

### **Prerequisites:**

- Knowledge of C

**Note:-** The complete C programs involved in this note are not study in detailed for the point of exam view these are only for practical purpose.

- For point of exam view writing algorithms and functions are important

---

## **Unit 1:**

### **Concept and definition of data structures**

- Information and its meaning
- Array in C
- The array as an ADT
- One dimensional array
- Two dimensional array
- Multidimensional array
- Structure
- Union
- Pointer

---

### **What is data structure?**

- Data structure is a way of organizing all data items and establishing relationship among those data items.
- Data structures are the building blocks of a program.  
Data structure mainly specifies the following four things:
  - Organization of data.
  - Accessing methods
  - Degree of associativity
  - Processing alternatives for information

To develop a program of an algorithm, we should select an appropriate data structure for that algorithm. Therefore algorithm and its associated data structures form a program.

**Algorithm + Data structure = Program**

A **static data structure** is one whose capacity is fixed at creation. For example, array. A **dynamic data structure** is one whose capacity is variable, so it can expand or contract at any time. For example, linked list, binary tree etc.

### **Abstract Data Types (ADTs)**

An **abstract data type** is a data type whose representation is hidden from, and of no concern to the application code. For example, when writing application code, we don't care how strings are represented: we just declare variables of type *String*, and manipulate them by using string operations.

Once an abstract data type has been designed, the programmer responsible for implementing that type is concerned only with choosing a suitable data structure and coding up the methods. On the other hand, application programmers are concerned only with using that type and calling its methods without worrying much about how the type is implemented.

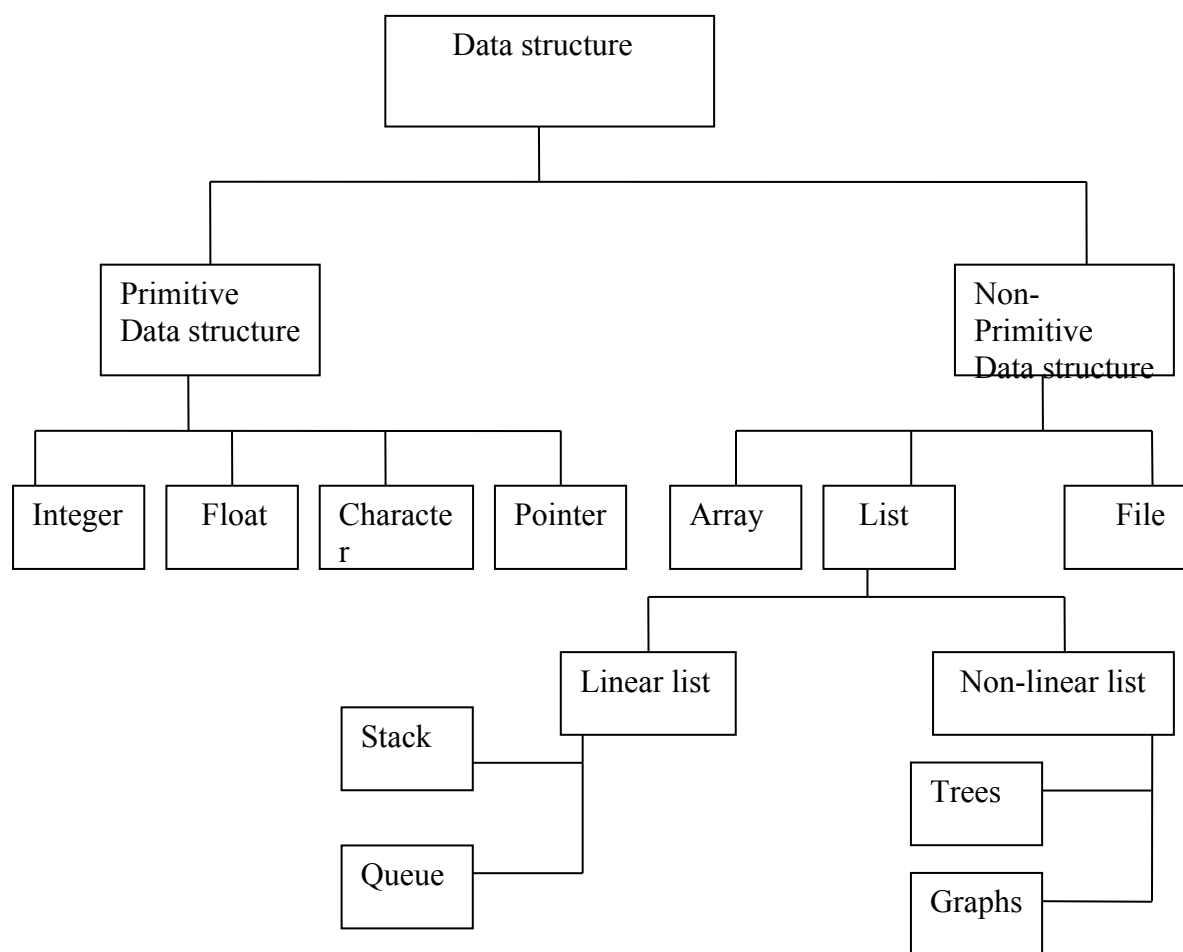
**Classification of data structure:**

Fig:- Classification of data structure

**Array**

- Array is a group of same type of variables that have common name
- Each item in the group is called an *element* of the array
- Each element is distinguished from another by an *index*
- All elements are stored contiguously in memory
- The elements of the array can be of any valid type- integers, characters, floating-point types or user-defined types

**Types of Array:****1). One dimensional array:**

The elements of the array can be represented either as a single column or as a single row.

**Declaring one-dimensional array:**

```
data_type array_name[size];
```

Following are some valid array declarations:

```
int age[15];
float weight[50];
int marks[100];
```

```
char section[12];
char name[10];
```

Following are some invalid array declarations in c:

```
int value[0];
int marks[0.5];
int number[-5];
```

### **Array Initialization (1-D):**

The general format of array initialization is:

```
data_type array_name[size]={element1,element2,.....,element n};
```

**for example:**

```
int age[5]={22,33,43,24,55};
int weight[]={55,6,77,5,45,88,96,11,44,32};
float a[]={2,3.5,7.9,-5.9,-8};
char section[4]={'A','B','C','D'};
char name[10]="Bhupendra";
```

**Example 1: A program to read n numbers and to find the sum and average of those numbers.**

```
#include<stdio.h>
void main()
{
    int a[100], i, n, sum=0;
    float avg;
    printf("Enter number of elements");
    scanf("%d",&n);
    printf("Enter %d numbers",n);
    for(i=0;i<n;i++)
    {
        scanf("%d",&a[i]);
        sum=sum+a[i];
        //sum+=a[i];
    }
    avg=sum/n;
    printf("sum=%d\n Average=%f", sum, avg);
}
```

### **Some common operations performed in one-dimensional array are:**

- Creating of an array
- Inserting new element at required position
- Deletion of any element
- Modification of any element
- Traversing of an array
- Merging of arrays

### **Insertion of new element at required position:**

Let we have an array

```
a[6]={1,5,7,6,22,90};
```

Suppose we want to insert 20 in array a, at location with index 4, it means the elements 22 and 90 must shift 1 position downwards as follows.

a[0]	1
a[1]	5
a[2]	7
a[3]	6
a[4]	22
a[5]	90
a[6]	0
a[7]	0
a[8]	0
a[9]	0

Original array

a[0]	1
a[1]	5
a[2]	7
a[3]	6
a[4]	.....
a[5]	22
a[6]	9 0
a[7]	0
a[8]	0

Elements shifted downwards

a[0]	1
a[1]	5
a[2]	7
a[3]	6
a[4]	20
a[5]	22
a[6]	90
a[7]	0
a[8]	0
a[9]	0

Array after insertion

**C- code for above problem:**

```
#include<stdio.h>
#include<conio.h>
void main()
{
    int a[100], pos, nel, i;
    clrscr();
    printf("Enter no of elements to be inserted");
    scanf("%d", &n);
    printf("Enter %d elements", n);
    for(i=0; i<n; i++)
    {
        scanf("%d", &a[i]);
    }
    printf("Enter position at which you want to insert new element");
    scanf("%d", &pos);
    printf("Enter new element");
    scanf("%d", &nel);
    for(i=n-1; i>=pos; i--)
    {
        a[i+1] = a[i];
    }
    a[pos]=nel;
    n++;
    printf("New array is:\n");
    for(i=0; i<n; i++)
    {
        printf("%d\t", a[i]);
    }
    getch();
}
```

**Deletion of any element from an array:**

Suppose we want to delete the element  $a[5]=90$ , then the elements following it were moved upward by one location as shown in fig. below:

a[0]	1
a[1]	5
a[2]	7
a[3]	6
a[4]	22
a[5]	90
a[6]	30
a[7]	10
a[8]	50
a[9]	8

a[0]	1
a[1]	5
a[2]	7
a[3]	6
a[4]	22
a[5]	.....
a[6]	30
a[7]	10
a[8]	50
a[9]	8

a[0]	1
a[1]	5
a[2]	7
a[3]	6
a[4]	22
a[5]	30
a[6]	10
a[7]	50
a[8]	8
a[9]	

Fig: - Deleting an element from one-dimensional array

**C- code for above problem:**

```
#include<stdio.h>
#include<conio.h>
void main()
{
    int a[100], pos, i;
    clrscr();
    printf("Enter no of elements to be inserted");
    scanf("%d", &n);
    printf("Enter %d elements", n);
    for(i=0; i<n; i++)
    {
        scanf("%d", &a[i]);
    }
    printf("Enter position at which you want to delete an element");
    scanf("%d", &pos);
    for(i=pos; i<n; i++)
    {
        a[i] = a[i+1];
    }
    n--;
    printf("New array is:\n");
    for(i=0; i<n; i++)
    {
        printf("%d\t", a[i]);
    }
}
```

**Modification of any element:**

```
#include<stdio.h>
#include<conio.h>
void main()
{
    int a[100], pos, nel, i;
    clrscr();
    printf("Enter no of elements to be inserted");
    scanf("%d", &n);
    printf("Enter %d elements", n);
    for(i=0;i<n;i++)
    {
        scanf("%d", &a[i]);
    }
    printf("Enter position at which you want to modify an element");
    scanf("%d", &pos);
    printf("Enter new element");
    scanf("%d", &nel);
    a[pos]=nel;
    printf("New array is:\n");
    for(i=0; i<n; i++)
    {
        printf("%d\t", a[i]);
    }
    getch();
}
```

**Traversing of an array:**

Traversing means to access all the elements of the array, starting from first element upto the last element in the array one-by-one.

**C- code for above problem:**

```
#include<stdio.h>
#include<conio.h>
void main()
{
    int a[100], pos, i;
    clrscr();
    printf("Enter no of elements to be inserted");
    scanf("%d", &n);
    printf("Enter %d elements", n);
    for(i=0;i<n;i++)
    {
        scanf("%d", &a[i]);
    }
    printf("Traversing of the array:\n");
    for(i=0; i<n; i++)
    {
        printf("%d\t", a[i]);
    }
    getch();
}
```

**Merging of two arrays:**

Merging means combining elements of two arrays to form a new array. Simplest way of merging two arrays is the first copy all elements of one array into a third empty array, and then copy all the elements of other array into third array.

Suppose we want to merge two arrays a[6] and b[4]. The new array says c will be having  $(6+4)=10$  elements as shown in figure below.

a[0]	1	b[0]	20	c[0]	1
a[1]	5	b[1]	25	c[1]	5
a[2]	7	b[2]	30	c[2]	7
a[3]	6	b[3]	35	c[3]	6
a[4]	12			c[4]	12
a[5]	15			c[5]	15
				c[6]	20
				c[7]	25
				c[8]	30
				c[9]	35

Fig: - Merging of two arrays

**C- code for above problem:**

```
#include<stdio.h>
#include<conio.h>
void main()
{
    int a[6], b[4], c[10], i, j;
    clrscr();
    printf("Enter elements of first array\n");
    for(i=0; i<6; i++)
        scanf("%d", &a[i]);
    printf("Enter elements of second array\n");
    for(i=0; i<4; i++)
    {
        scanf("%d", &a[i]);
    }
    for(i=0; i<6; i++)
    {
        c[i]=a[i];
    }
    j=i; // here i=j=6
    for(i=0; i<4; i++)
    {
        c[j]=a[i];
        j++;
    }
    printf("The resulting array is:\n");
    for(i=0; i<10; i++)
    {
        printf("%d\t", c[i]);
    }
}
```

```

    getch();
}

```

## **Two-Dimensional array:**

When we declare two dimensional array, the first subscript written is for the number of rows and the second one is for the column.

### **Declaration of 2- D array:**

Return\_type array\_name[row\_size][column\_size];

### **Example;**

```
int a[3][4];
```

```
float b[10][10];
```

int this first example, 3 represents number of rows and 4 represents number of columns.

- Think, two-dimensional arrays as tables/matrices arranged in rows and columns
- Use first subscript to specify row no and the second subscript to specify column no.

	Column 0	Column 1	Column 2	Column 3
Row 0	a[0][0]	a[0][1]	a[0][2]	a[0][3]
Row 1	a[1][0]	a[1][1]	a[1][2]	a[1][3]
Row 2	a[2][0]	a[2][1]	a[2][2]	a[2][3]

Array name
Row subscript
Column subscript

## **Array Initialization (2-D):**

The general format of array initialization is:

**data\_type array\_name[row\_size][col\_size]={element1,element2,.....,element n};**

### **for example:**

```
int a[2][3]={33,44,23,56,77,87};
```

**or**

```
int a[2][3]={ {33,44,23},
              {56, 77, 87} };
```

### **Example 1: A program to find addition of any two matrices by using function**

```

#include<stdio.h>
#include<conio.h>
void display(int [][], int, int); //function prototype
void main()
{
    int a[10][10], b[10][10], c[10][10], i, j, r, c;
    clrscr();
    printf("Enter size of a matrix");
    scanf("%d%d", &r, &c);
    printf("Enter elements of first matrix\n");
    for(i=0; i<r; i++)
    {

```



```

        for(j=0;j<c;j++)
        {
            scanf("%d",&a[i][j]);
        }
    }
    printf("Enter elements of second matrix\n");
    for(i=0;i<r;i++)
    {
        for(j=0;j<c;j++)
        {
            scanf("%d",&b[i][j]);
        }
    }
    // finding sum
    for(i=0;i<r;i++)
    {
        for(j=0;j<c;j++)
        {
            c[i][j]=a[i][j]+b[i][j];
        }
    }
    printf("The first matrix is\n");
    display(a, r, c);           //function call
    printf("The second matrix is\n");
    display(b, r, c);           //function call
    printf("The resulting matrix is\n");
    display(c, r, c);           //function call
    getch();
}
void display(int d[10][10], int r, int c) //function definition
{
    int i, j;
    for(i=0;i<r;i++)
    {
        for(j=0;j<c;j++)
        {
            printf("%d\t", d[i][j]);
        }
        printf("\n");
    }
}

```

### Example 2: A program to find transposition of a matrix by using function

```

#include<stdio.h>
void display(int [], int, int); //function prototype
void main()
{
    int a[10][10], t[10][10], i, j, r, c;
    printf("Enter no of rows and no of columns");
    scanf("%d%d",&r,&c);
    printf("Enter elements of a matrix\n");
    for(i=0;i<r;i++)
    {
        for(j=0;j<c;j++)
        {
            scanf("%d",&a[i][j]);
        }
    }
}

```

```

    }
}
//finding transpose of a matrix
for(i=0;i<r;i++)
{
    for(j=0;j<c;j++)
    {
        t[i][j]=a[j][i];
    }
}
printf("the original matrix is\n");
display(a, r, c);          //function call
printf("the transposed matrix is\n");
display(t, r, c);          //function call
}
void display(int x[][], int r, int c)    //function definition
{
    int i, j;
    for(i=0;i<r;i++)
    {
        for(j=0;j<c;j++)
        {
            printf("%d\t",x [i][j]);
        }
        printf("\n");
    }
}

```

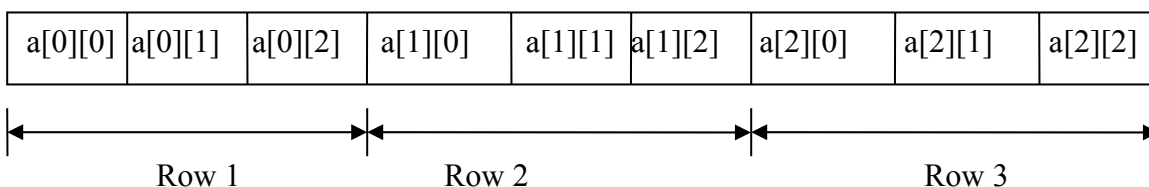
### **Implementation of a two-dimensional array:**

A two dimensional array can be implemented in a programming language in two ways:

- Row-major implementation
- Column-major implementation

### **Row-major implementation:**

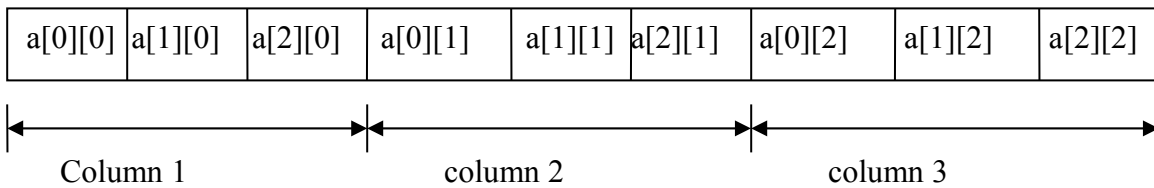
Row-major implementation is a linearization technique in which elements of array are reader from the keyboard row-wise i.e. the complete first row is stored, and then the complete second row is stored and so on. For example, an array `a[3][3]` is stored in the memory as shown in fig below:



### **Column-major implementation:**

In column major implementation memory allocation is done column by column i.e. at first the elements of the complete first column is stored, and then elements of

complete second column is stored, and so on. For example an array  $a[3][3]$  is stored in the memory as shown in the fig below:



### **Multi-Dimensional array**

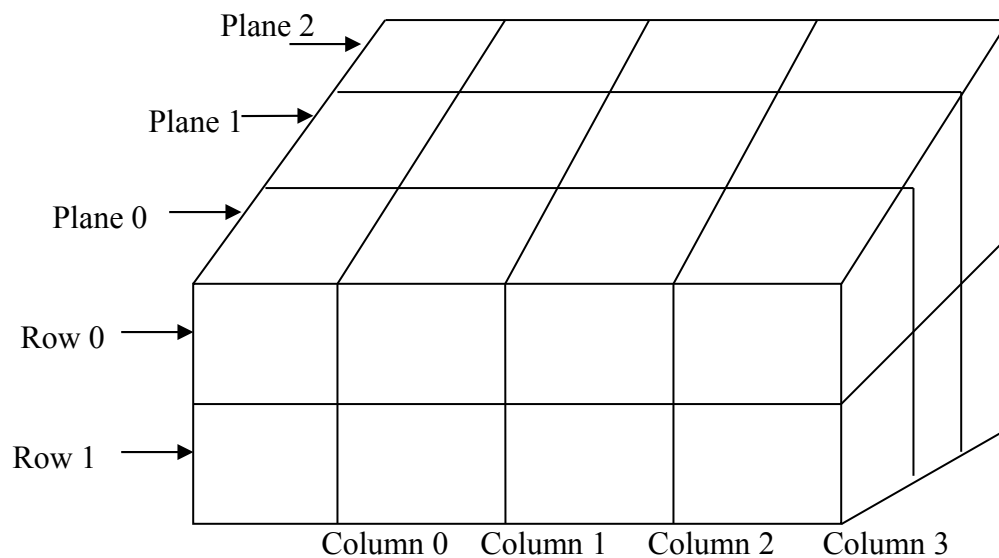
C also allows arrays with more than two dimensions. For example, a three dimensional array may be declared by

```
int a[3][2][4];
```

Here, the first subscript specifies a plane number, the second subscript a row number and the third a column number.

However C does allow an arbitrary number of dimensions. For example, a six-dimensional array may be declared by

```
int b[3][4][6][8][9][2];
```



### **Show that an array is an ADT:**

Let  $A$  be an array of type  $T$  and has  $n$  elements then it satisfied the following operations:

- **CREATE( $A$ ):** Create an array  $A$
- **INSERT( $A, X$ ):** Insert an element  $X$  into an array  $A$  in any location
- **DELETE( $A, X$ ):** Delete an element  $X$  from an array  $A$
- **MODIFY( $A, X, Y$ ):** modify element  $X$  by  $Y$  of an array  $A$
- **TRAVELS( $A$ ):** Access all elements of an array  $A$
- **MERGE( $A, B$ ):** Merging elements of  $A$  and  $B$  into a third array  $C$

Thus by using a one-dimensional array we can perform above operations thus an array acts as an ADT.

## **Structure:**

A structure is a collection of one or more variables, possibly of different types, grouped together under a single name.

An array is a data structure in which all the members are of the same data type. Structure is another data structure in which the individual elements can differ in type. Thus, a single structure might contain integer elements, floating-point elements and character elements. The individual structure elements are referred to as *members*.

Defining a structure: A structure is defined as

```
struct structure_name
{
    member 1;
    member 2;
    .....
    member n;
};
```

We can define a structure to hold the information of a student as follows:

```
struct Student
{
    char name[2];
    int roll;
    char sec;
    float marks;
};
```

## **Structure variable declaration:**

```
struct Student s1, s2, s3;
```

We can combine both template declaration and structure variable declaration in one statement.

Eg,

```
struct Student
{
    char name[2];
    int roll;
    char sec;
    float marks;
} s1, s2, s3;
```

**Accessing members of a structure:**

There are two types of operators to access members of a structure. Which are:

- Member operator (dot operator or period operator (.))
- Structure pointer operator (->).

**Structure initialization:**

Like any data type, a structure variable can be initialized as follows:

**struct** Student

```
{
    char name[20];
    int roll;
    char sec;
    float marks;
};
```

**struct** Student s1={"Raju", 22, 'A', 55.5};

The s1 is a structure variable of type Student, whose members are assigned initial values. The first member (name[20]) is assigned the string "Raju", the second member (roll) is assigned the integer value 22, the third member (sec) is assigned the character 'A', and the fourth member (marks) is assigned the float value 55.5.

**Example: program illustrates the structure in which read member elements of structure and display them.**

```
#include<stdio.h>
```

```
void main()
```

```
{
    struct Student
    {
        char name[20];
        int roll;
        char sec;
        float marks;
    };
    struct Student s1;
    clrscr();
    printf("Enter the name of a student");
    gets(s1.name);
    printf("Enter the roll number of a student");
    scanf("%d",&s1.roll);
    printf("Enter the section of a student");
    scanf("%c",&s1.sec);
    printf("Enter the marks obtained by the student");
    scanf("%f",&s1.marks);
    //displaying the records
    printf("Name=%s\n Roll number =%d\n Section=%c\n Obtained marks=%f",s1.name,
    s1.roll, s1.sec, s1.marks);
}
```

**Structures within Structures:**

Structures within structures mean nesting of structures. Study the following example and understand the concepts.

**Example:** the following example shows a structure definition having another structure as a member. In this example, person and students are two structures. Person is used as a member of student.(person within Student)

```
#include<stdio.h>
struct Prsrson
{
    char name[20];
    int age;
};
struct Student
{
    int roll;
    char sec;
    struct Person p;
};
void main()
{
    struct Student s;
    printf("Enter the name of a student");
    gets(s.p.name);
    printf("Enter age");
    scanf("%d",&s.p.age);
    printf("Enter the roll number of a student");
    scanf("%d",&s.roll);
    printf("Enter the section of a student");
    scanf("%c",&s.sec);
    //displaying the records
    printf("Name=%s\n Roll number =%d\n Age=%d\n Section=%c\n",s.p.name, s.roll,
    s.p.age, s.sec);
}
```

**Equivalent form of nested structure is:**

```
struct Student
{
    int roll;
    char sec;
    struct Person
    {
        char name[20];
        int age;
    }p;
};
```

### **Passing entire structures to functions:**

```
#include<stdio.h>
void display(struct student);
struct student
{
    char name[20];
    int age;
    int roll;
    char sec;
};
void main()
{
    struct student s;
    int i;
    printf("Enter the name of a student");
    gets(s.name);
    printf("Enter age");
    scanf("%d",&s.age);
    printf("Enter the roll number of a student");
    scanf("%d",&s.roll);
    printf("Enter the section of a student");
    scanf("%c",&s.sec);
    display(s); //function call
}
```

```

}
void sisplay(struct student st)
{
    //displaying the records
    printf("Name=%s\n Roll number =%d\n Age=%d\n Section=%c\n",st.name, st.roll,
    st.age, st.sec);
}

```

## **Unions:**

Both **structure** and **unions** are used to group a number of different variables together. Syntactically both structure and unions are exactly same. The main difference between them is in storage. In structures, each member has its own memory location but all members of union use the same memory location which is equal to the greatest member's size.

### **Declaration of union:**

The general syntax for declaring a union is:

```

union union_name
{
    data_type    member1;
    data_type    member2;
    data_type    member3;
    .....
    .....
    data_type    memberN;
};

```

We can define a **union** to hold the information of a student as follows:

```

union Student
{
    char name[2];
    int roll;
    char sec;
    float marks;
};

```

### **union variable declaration:**

```
union Student s1, s2, s3;
```

we can combine both template declaration and **union** variable declaration in one statement.

Eg,

```

union Student
{
    char name[2];
    int roll;
    char sec;
    float marks;
} s1, s2, s3;

```

**Differences between structure and unions**

1. The amount of memory required to store a structure variable is the sum of sizes of all the members. On the other hand, in case of a union, the amount of memory required is the same as member that occupies largest memory.

```
#include<stdio.h>
#include<conio.h>
struct student
{
    int roll_no;
    char name[20];
};
union employee
{
    int ID;
    char name[20];
};
void main()
{
    struct student s;
    union employee e;
    printf("\nsize of s = %d bytes",sizeof(s)); // prints 22 bytes
    printf("\nSize of e = %d bytes",sizeof(e)); // prints 20 bytes
    getch();
}
```

**Pointers**

A pointer is a variable that holds address (memory location) of another variable rather than actual value. Also, a pointer is a variable that points to or references a memory location in which data is stored. Each memory cell in the computer has an address that can be used to access that location. So, a pointer variable points to a memory location and we can access and change the contents of this memory location via the pointer. Pointers are used frequently in C, as they have a number of useful applications. In particular, pointers provide a way to return multiple data items from a function via function arguments.

**Pointer Declaration**

Pointer variables, like all other variables, must be declared before they may be used in a C program. We use asterisk (\*) to do so. Its general form is:

*ata-type \*ptrvar;*

**For example,**

*int\* ptr;*

*float \*q;*

*char \*r;*



This statement declares the variable *ptr* as a pointer to *int*, that is, *ptr* can hold address of an integer variable.

### **Reasons for using pointer:**

- A pointer enables us to access a variable that is defined outside the function.
- Pointers are used in dynamic memory allocation.
- They are used to pass array to functions.
- They produce compact, efficient and powerful code with high execution speed.
- The pointers are more efficient in handling the data table.
- They use array of pointers in character strings result in saving of data storage space in memory. Sorting strings using pointer is very efficient.
- With the help of pointer, variables can be swapped without physically moving them.
- Pointers are closely associated with arrays and therefore provides an alternate way to access individual array elements.

### **Pointer initialization:**

Once a pointer variable has been declared, it can be made to point to a variable using an assignment statement as follows:

```
int marks;
int *marks_pointer;
Marks_pointer=&marks;
```

### **Passing (call) by Value and Passing (call) by Reference**

Arguments can generally be passed to functions in one of the two ways:

- Sending the values of the arguments (pass by value)
- Sending the addresses of the arguments (pass by reference)

**Pass by value:** In this method, the value of each of the actual arguments in the calling function is copied into corresponding formal arguments of the called function. With this method the changes made to the formal arguments in the called function have no effect on the values of actual arguments in the calling function. The following program illustrates 'call by value'.

```
#include<stdio.h>
#include<conio.h>
void main()
{
    int a,b;
    void swap(int, int );
    clrscr();
    a = 10;
    b = 20;
    swap(a,b);
    printf("a = %d\tb = %d",a,b);
    getch();
}
void swap(int x, int y)
{
    int t;
```

```

    t = x;
    x = y;
    y = t;
    printf("x = %d\ty = %d\n",x,y);
}

```

The output of the above program would be

$x = 20$   $y = 10$

$a = 10$   $b = 20$

Note that values of **a** and **b** are unchanged even after exchanging the values of x and y.

**Pass by reference:** In this method, the addresses of actual arguments in the calling function are copied into formal arguments of the called function. This means that using these addresses we would have an access to the actual arguments and hence we would be able to manipulate them. The following program illustrates this fact.

```

#include<stdio.h>
#include<conio.h>
void main()
{
    int a,b;
    void swap(int*, int*);
    clrscr();
    a = 10;
    b = 20;
    swap(&a,&b);
    printf("a = %d\tb = %d",a,b);
    getch();
}
void swap(int *x, int *y)
{
    int t;
    t = *x;
    *x = *y;
    *y = t;
    printf("x = %d\ty = %d\n", *x, *y);
}

```

The output of the above program would be

$x = 20$   $y = 10$

$a = 20$   $b = 10$

**Note:** We can use call by reference to return multiple values from the function.

## **Pointers and Arrays**

An array name by itself is an address, or pointer. A pointer variable can take different addresses as values. In contrast, an array name is an address, or pointer, that is fixed.

### **Pointers and One-dimensional Arrays**

In case of one dimensional array, an array name is really a pointer to the first element in the array. Therefore, if x is a one-dimensional array, then the address of the first array element can be expressed as either  $\&x[0]$  or simply x. Moreover, the address of the

second array element can be expressed as either `&x[1]` or as `(x+1)`, and so on. In general, the address of array element `(x+i)` can be expressed as either `&x[i]` or as `(x+i)`. Thus we have two different ways to write the address of any array element: we can write the actual array element, preceded by an ampersand; or we can write an expression in which the subscript is added to the array name.

Since, `&x[i]` and `(x+i)` both represent the address of the *i*th element of *x*, it would seem reasonable that `x[i]` and `*(x+i)` both represent the contents of that address, i.e., the value of the *i*th element of *x*. The two terms are interchangeable. Hence, either term can be used in any particular situation. The choice depends upon your individual preferences. For example,

***/\* Program to read n numbers in an array and display their sum and average \*/***

```
#include<stdio.h>
#include<conio.h>
#define SIZE 100
void main()
{
    float a[SIZE],sum=0,avg;
    int n,i;
    clrscr();
    printf("How many numbers?");
    scanf("%d",&n);
    printf("Enter numbers:\n");
    for(i=0;i<n;i++)
    {
        scanf("%f",a+i); // scanf("%f",&a[i]);
        sum=sum+*(a+i); //sum=sum+a[i];
    }
    avg=sum/n;
    printf("Sum=%f\n",sum);
    printf("Average=%f",avg);
    getch();
}
```

***/\* using pointer write a program to add two 3 × 2 matrices and print the result in matrix form \*/***

```
#include<stdio.h>
#include<conio.h>
#define ROW 3
#define COL 2
void main()
{
    int a[ROW][COL],b[ROW][COL],i,j,sum;
    clrscr();
    printf("Enter elements of first matrix:\n");
    for(i=0;i<ROW;i++)
    {
        for(j=0;j<COL;j++)
            scanf("%d", (*(a+i)+j));
        printf("\n");
    }
}
```

```

printf("Enter elements of second matrix:\n");
for(i=0;i<ROW;i++)
{
    for(j=0;j<COL;j++)
        scanf("%d", (*(b+i)+j));
    printf("\n");
}
printf("Addition matrix is:\n");
for(i=0;i<ROW;i++)
{
    for(j=0;j<COL;j++)
    {
        sum = (*(a+i)+j)+(*(b+i)+j);
        printf("%d\t",sum);
    }
    printf("\n");
}
getch();
}

```

**/\*Sum of two matrix using dynamic memory allocation\*/**

```

#include<stdio.h>
#include<conio.h>
#include<stdlib.h>
void read(int**, int, int);
void write(int**, int, int);
void main()
{
    int **a;
    int **b;
    int **s;
    int r,c,i,j;
    clrscr();
    printf("Enter no of row and columns of a matrix\n");
    scanf("%d%d",&r,&c);
    for(i=0;i<r;i++)
    {
        *(a+i)=(int*)malloc(sizeof(int)*c);
        *(b+i)=(int*)malloc(sizeof(int)*c);
        *(s+i)=(int*)malloc(sizeof(int)*c);
    }
    printf("Enter elements of first matrix");
    read(a,r,c);
    printf("Enter elements of Second matrix");
    read(b,r,c);
    for(i=0;i<r;i++)
    {
        for(j=0;j<c;j++)
        {
            (*(s+i)+j)=(*(a+i)+j)+(*(b+i)+j);
        }
    }
    printf("Matrix A is:\n\n");
    write(a,r,c);
    printf("Matrix B is:\n\n");
}

```

```

        write(b,r,c);
        printf("Sum of matrix A and B is:\n\n");
        write(s,r,c);
        getch();
    }
    void read(int **x,int r,int c)
    {
        int i,j;
        for(i=0;i<r;i++)
        {
            for(j=0;j<c;j++)
            {
                scanf("%d",&(x[i][j]));
            }
        }
    }
    void write(int **y,int r,int c)
    {
        int i,j;
        for(i=0;i<r;i++)
        {
            for(j=0;j<c;j++)
            {
                printf("%d\t",&(y[i][j]));
            }
            printf("\n");
        }
    }
}

```

### **Lab work No:1**

***#Write a program using user defined functions to sum two 2-dimensional arrays and store the sum of the corresponding elements into third array and print all the three arrays with its values at the corresponding places (not in a single row), get the transpose of the third matrix and print***

```

#include<stdio.h>
#include<conio.h>
void read(int [10][10],char,int,int);
void write(int [10][10],int,int);
void main()
{
    int a[10][10];
    int b[10][10];
    int s[10][10];
    int t[10][10];
    int r,c,i,j;
    clrscr();
    printf("Enter no of row and columns of a matrix\n");
    scanf("%d%d",&r,&c);
    read(a,'a',r,c);
    read(b,'b',r,c);
    for(i=0;i<r;i++)
    {
        for(j=0;j<c;j++)
        {
            s[i][j]=a[i][j]+b[i][j];

```

```

    }
}
printf("Matrix A is:\n\n");
write(a,r,c);
printf("Matrix B is:\n\n");
write(b,r,c);
printf("Sum of matrix A and B is:\n\n");
write(s,r,c);
for(i=0;i<r;i++)
{
    for(j=0;j<c;j++)
    {
        t[i][j]=s[j][i];
    }
}
printf("Transpose of resultant matrix is:\n");
write(t,r,c);
getch();
}

```

```

void read(int x[10][10],char ch,int r,int c)
{
    int i,j;
    printf("Enter elements of a matrix[%c]\n",ch);
    for(i=0;i<r;i++)
    {
        for(j=0;j<c;j++)
        {
            scanf("%d",&x[i][j]);
        }
    }
}

```

```

void write(int y[10][10],int r,int c)
{
    int i,j;
    for(i=0;i<r;i++)
    {
        for(j=0;j<c;j++)
        {
            printf("%d\t",y[i][j]);
        }
        printf("\n");
    }
}

```

### **Lab work No:2**

***Write a program using an array to perform the following tasks:(use switch case for menu)***

- a. Insert element into an array at specified position***
- b. Delete element from an array***
- c. Traversing***
- d. Searching a particular element in the array***

```
#include<stdio.h>
```

```

#include<conio.h>
void insert(int [100], int*);
void delet(int [100], int*);
void traverse(int [100], int*);
void searching(int [100], int*);
void main()
{
    int a[100], nel, pos, i;
    int n;
    int choice;
    clrscr();
    printf("Enter no of elements to be inserted");
    scanf("%d", &n);
    printf("Enter %d elements", n);
    for(i=0; i<n; i++)
    {
        scanf("%d", &a[i]);
    }
    do
    {
        printf("\nmanu for program:\n");
        printf("1:insert\n2:delete\n3:Traverse\n4:searching\n5:exit\n");
        printf("Enter your choice\n");
        scanf("%d",&choice);
        switch(choice)
        {
            case 1:
                insert(a,&n);
                break;
            case 2:
                delet(a,&n);
                break;
            case 3:
                traverse(a,&n);
                break;
            case 4:
                searching(a,&n);
                break;
            case 5:
                exit(1);
                break;
            default:
                printf("Invalid choice");
        }
    } while(choice<6);
}

void insert(int a[100], int *n)
{
    int pos, nel, i;
    printf("Enter position at which you want to insert new element");
    scanf("%d", &pos);
    printf("Enter new element");
    scanf("%d", &nel);
    for(i=*n-1; i>=pos; i--)
    {

```

```

        a[i+1] = a[i];
    }
    a[pos]=nel;
    *n=*n+1;
    printf("New array is:\n");
    for(i=0; i<*n; i++)
    {
        printf("%d\t", a[i]);
    }
}

void delet(int a[100], int *n)
{
    int pos, i;
    printf("Enter position at which you want to delete an element");
    scanf("%d", &pos);
    for(i=pos; i<*n; i++)
    {
        a[i] = a[i+1];
    }
    *n=*n-1;
    printf("New array is:\n");
    for(i=0; i<*n; i++)
    {
        printf("%d\t", a[i]);
    }
}

void traverse(int a[100], int *n)
{
    int i;
    printf("Elements of array are:\n");
    for(i=0; i<*n; i++)
    {
        printf("%d\t", a[i]);
    }
}

void searching(int a[100], int *n)
{
    int k, i;
    printf("Enter searched item");
    scanf("%d", &k);
    for(i=0; i<*n; i++)
    {
        if(k==a[i])
        {
            printf("*****successful search*****");
            break;
        }
    }
    if(i==*n)
    printf("*****unsuccessful search*****");
}

```



**Lab work No:3*****Write a program to multiply two  $m \times n$  matrices using dynamic memory allocation.******Hint  $c[i][j] += a[i][k] * b[k][j]$ ;***

```

#include<stdio.h>
#include<conio.h>
#include<stdlib.h>
void read(int**,int,int);
void write(int**,int,int);
void main()
{
    int **a;
    int **b;
    int **m;
    int r1,c1,r2,c2,i,j,k;
    clrscr();
    printf("Enter no of row and columns of first matrix\n");
    scanf("%d%d",&r1,&c1);
    printf("Enter no of row and columns of second matrix\n");
    scanf("%d%d",&r2,&c2);
    for(i=0;i<r1;i++)
    {
        *(a+i)=(int*)malloc(sizeof(int)*c1);
        *(m+i)=(int*)malloc(sizeof(int)*c2);
    }
    for(i=0;i<r2;i++)
    {
        *(b+i)=(int*)malloc(sizeof(int)*c2);

    }
    printf("Enter elements of first matrix");
    read(a,r1,c1);
    printf("Enter elements of Second matrix\n");
    read(b,r2,c2);
    for(i=0;i<r1;i++)
    {
        for(j=0;j<c2;j++)
        {
            (*(m+i)+j)=0;
            for(k=0;k<c1;k++)
            {
                (*(m+i)+j)+=(*(a+i)+k)*(*(b+k)+j));
            }
        }
    }
    printf("Matrix A is:\n\n");
    write(a,r1,c1);
    printf("Matrix B is:\n\n");
    write(b,r2,c2);
    printf("product of matrix A and B is:\n\n");
    write(m,r1,c2);
    getch();
}
void read(int **x,int r,int c)
{

```

```

int i,j;
for(i=0;i<r;i++)
{
    for(j=0;j<c;j++)
    {
        scanf("%d",&(x+i+j));
    }
}

void write(int**y,int r,int c)
{
    int i,j;
    for(i=0;i<r;i++)
    {
        for(j=0;j<c;j++)
        {
            printf("%d\t",&*(y+i+j));
        }
        printf("\n");
    }
}

```

## Unit 2

### Algorithm

- Concept and definition
- Design of algorithm
- Characteristic of algorithm
- Big O notation

#### Algorithm:

An algorithm is a precise specification of a sequence of instructions to be carried out in order to solve a given problem. Each instruction tells what task is to be done. There should be a finite number of instructions in an algorithm and each instruction should be executed in a finite amount of time.

#### Properties of Algorithms:

- **Input:** A number of quantities are provided to an algorithm initially before the algorithm begins. These quantities are inputs which are processed by the algorithm.
- **Definiteness:** Each step must be clear and unambiguous.
- **Effectiveness:** Each step must be carried out in finite time.
- **Finiteness:** Algorithms must terminate after finite time or step
- **Output:** An algorithm must have output.

- **Correctness:** Correct set of output values must be produced from the each set of inputs.

**Write an algorithm to find the greatest number among three numbers:**

Step 1: Read three numbers and store them in X, Y and Z

Step 2: Compare X and Y. if X is greater than Y then go to step 5 else step 3

Step 3: Compare Y and Z. if Y is greater than Z then print “Y is greatest” and go to step 7 otherwise go to step 4

Step 4: Print “Z is greatest” and go to step 7

Step 5: Compare X and Z. if X is greater than Z then print “X is greatest” and go to step 7 otherwise go to step 6

Step 6: Print “Z is greatest” and go to step 7

Step 7: Stop

**Big Oh (O) notation:**

When we have only asymptotic upper bound then we use O notation. A function  $f(x)=O(g(x))$  (read as f(x) is big oh of g(x) ) iff there exists two positive constants c and  $x_0$  such that

for all  $x \geq x_0$ ,  $f(x) \leq c \cdot g(x)$

The above relation says that g(x) is an upper bound of f(x)

***O(1) is used to denote constants.***

***Example:***

$f(x)=5x^3+3x^2+4$  find big oh(O) of f(x)

solution:  $f(x)= 5x^3+3x^2+4 \leq 5x^3+3x^3+4x^3$  if  $x>0$   
 $\leq 12x^3$

$\Rightarrow f(x) \leq c \cdot g(x)$

where  $c=12$  and  $g(x)=x^3$

Thus by definition of big oh  $O(f(x))=O(x^3)$

**Big Omega ( $\Omega$ ) notation:**

Big omega notation gives asymptotic lower bound. A function  $f(x)=\Omega(g(x))$  (read as g(x) is big omega of g(x) ) iff there exists two positive constants c and  $x_0$  such that for all  $x \geq x_0$ ,  $0 \leq c \cdot g(x) \leq f(x)$ .

The above relation says that g(x) is a lower bound of f(x).

**Big Theta ( $\Theta$ ) notation:**

When we need asymptotically tight bound then we use notation. A function  $f(x) = (g(x))$  (read as  $f(x)$  is big theta of  $g(x)$ ) iff there exists three positive constants  $c_1$ ,  $c_2$  and  $x_0$  such that for all  $x \geq x_0$ ,  $c_1 * g(x) \leq f(x) \leq c_2 * g(x)$

The above relation says that  $f(x)$  is order of  $g(x)$

**Example:**

$$f(n) = 3n^2 + 4n + 7$$

$g(n) = n^2$ , then prove that  $f(n) = (g(n))$ .

**Proof:** let us choose  $c_1$ ,  $c_2$  and  $n_0$  values as 14, 1 and 1 respectively then we can have,

$$f(n) \leq c_1 * g(n), n \geq n_0 \text{ as } 3n^2 + 4n + 7 \leq 14 * n^2, \text{ and}$$

$$f(n) \geq c_2 * g(n), n \geq n_0 \text{ as } 3n^2 + 4n + 7 \geq 1 * n^2$$

for all  $n \geq 1$  (in both cases).

So  $c_2 * g(n) \leq f(n) \leq c_1 * g(n)$  is trivial.

Hence  $f(n) = Q(g(n))$ .

**Example : Fibonacci Numbers**

**Input:**  $n$

**Output:**  $n^{\text{th}}$  Fibonacci number.

**Algorithm:** assume  $a$  as first(previous) and  $b$  as second(current) numbers

fib(n)

```
{
    a = 0, b = 1, f = 1 ;
    for(i = 2 ; i <= n ; i++)
    {
        f = a+b ;
        a=b ;
        b=f ;
    }
    return f ;
}
```

**Efficiency:**

**Time Complexity:** The algorithm above iterates up to  $n-2$  times, so time complexity is  $O(n)$ .

**Space Complexity:** The space complexity is constant i.e.  $O(1)$ .

**Example : Bubble sort****Algorithm**

BubbleSort(A, n)

```
{
    for(i = 0; i < n-1; i++)
    {
        for(j = 0; j < n-i-1; j++)
        {
            if(A[j] > A[j+1])
            {
```

```
temp = A[j];
A[j] = A[j+1];
A[j+1] = temp;
```

```
}
```

```
}
```

```
}
```

```
}
```

**Time Complexity:**

Inner loop executes for (n-1) times when i=0, (n-2) times when i=1 and so on:

$$\begin{aligned}\text{Time complexity} &= (n-1) + (n-2) + (n-3) + \dots + 2 + 1 \\ &= O(n^2)\end{aligned}$$

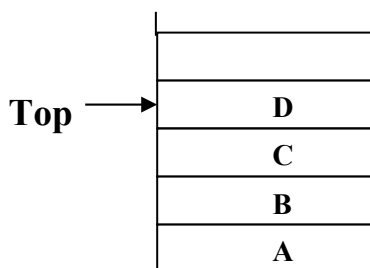
**Unit:-3****The stack**

- a. concept and definition
  - primitive operations
  - Stack as an ADT
  - Implementing PUSH and POP operation
  - Testing for overflow and underflow conditions
- b. The infix, postfix and prefix
  - Concept and definition
  - Evaluating the postfix operation
  - Converting from infix to postfix
- c. Recursion
  - Concept and definition
  - Implementation of:
    - ✓ Multiplication of natural numbers
    - ✓ Factorial
    - ✓ Fibonacci sequences
    - ✓ The tower of Hanoi

**Introduction to Stack**

*A stack is an ordered collection of items into which new items may be inserted and from which items may be deleted at one end, called the top of the stack.* The deletion and insertion in a stack is done from top of the stack.

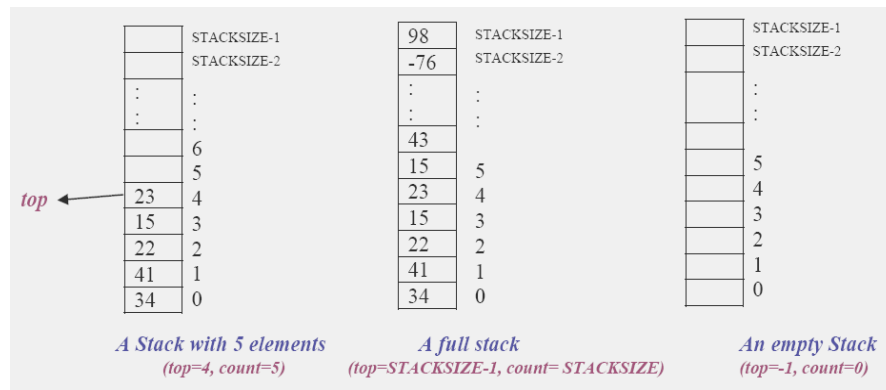
The following fig shows the stack containing items:



(Fig: A stack containing elements or items)

Intuitively, a stack is like a pile of plates where we can only (conveniently) remove a plate from the top and can only add a new plate on the top.

In computer science we commonly place numbers on a stack, or perhaps place records on the stack



### Applications of Stack:

Stack is used directly and indirectly in the following fields:

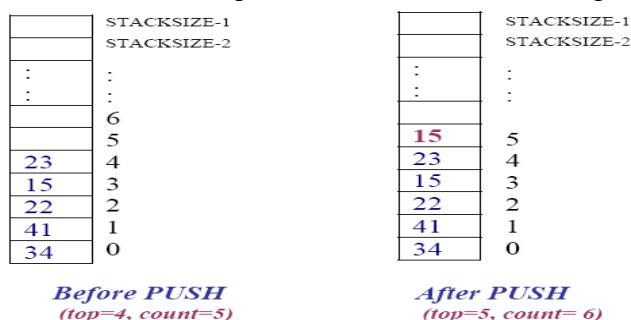
- To evaluate the expressions (postfix, prefix)
- To keep the page-visited history in a Web browser
- To perform the undo sequence in a text editor
- Used in recursion
- To pass the parameters between the functions in a C program
- Can be used as an auxiliary data structure for implementing algorithms
- Can be used as a component of other data structures

### Stack Operations:

The following operations can be performed on a stack:

❖ **PUSH operation:** The push operation is used to add (or push or insert) elements in a stack

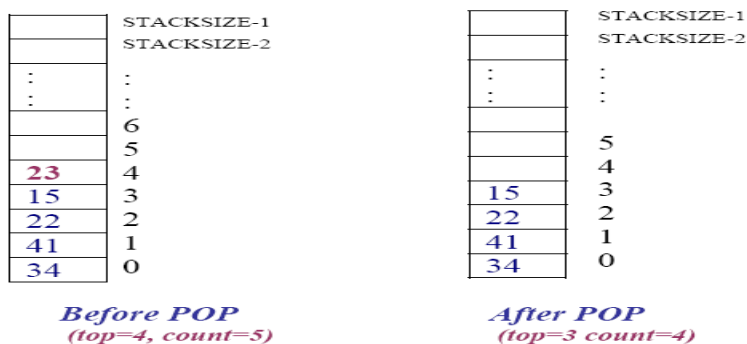
- ▶ When we add an item to a stack, we say that we **push** it onto the stack
- ▶ The last item put into the stack is at the top



❖ **POP operation:** The pop operation is used to remove or delete the top element from the stack.

- ▶ When we remove an item, we say that we **pop** it from the stack

- ▶ When an item popped, it is always the top item which is removed



The **PUSH** and the **POP** operations are the *basic or primitive* operations on a stack. Some others operations are:

- **CreateEmptyStack** operation: This operation is used to create an empty stack.
- **IsFull** operation: The isfull operation is used to check whether the stack is full or not ( i.e. stack overflow)
- **IsEmpty** operation: The isempty operation is used to check whether the stack is empty or not. (i. e. stack underflow)
- **Top operations**: This operation returns the current item at the top of the stack, it doesn't remove it

### The Stack ADT:

A **stack** of elements of type **T** is a finite sequence of elements of **T** together with the operations

- **CreateEmptyStack(S)**: Create or make stack **S** be an empty stack
- **Push(S, x)**: Insert **x** at one end of the stack, called its **top**
- **Top(S)**: If stack **S** is not empty; then retrieve the element at its **top**
- **Pop(S)**: If stack **S** is not empty; then delete the element at its **top**
- **IsFull(S)**: Determine if **S** is full or not. Return **true** if **S** is full stack; return **false** otherwise
- **IsEmpty(S)**: Determine if **S** is empty or not. Return **true** if **S** is an empty stack; return **false** otherwise.

### Implementation of Stack:

Stack can be implemented in two ways:

1. Array Implementation of stack (or static implementation)
2. Linked list implementation of stack (or dynamic)

### Array (static) implementation of a stack:

It is one of two ways to implement a stack that uses a one dimensional array to store the data. In this implementation top is an integer value (an index of an array) that indicates the top position of a stack. Each time data is added or removed, top is incremented or decremented accordingly, to keep track of current top of the stack. By convention, in C implementation the empty stack is indicated by setting the value of top to -1(top=-1).

*#define MAX 10*

```

struct stack
{
    int items[MAX]; //Declaring an array to store items
    int top;        //Top of a stack
};
typedef struct stack st;

```

### **Creating Empty stack :**

The value of top=-1 indicates the empty stack in C implementation.

```

/*Function to create an empty stack*/
void create_empty_stack(st *s)
{
    s->top=-1;
}

```

### **Stack Empty or Underflow:**

This is the situation when the stack contains no element. At this point the top of stack is present at the bottom of the stack. In array implementation of stack, conventionally top=-1 indicates the empty.

The following function return 1 if the stack is empty, 0 otherwise.

```

int isempty(st *s)
{
    if(s->top==-1)
        return 1;
    else
        return 0;
}

```

### **Stack Full or Overflow:**

This is the situation when the stack becomes full, and no more elements can be pushed onto the stack. At this point the stack top is present at the highest location (MAXSIZE-1) of the stack. The following function returns true (1) if stack is full false (0) otherwise.

```

int isfull(st *s)
{
    if(s->top==MAX-1)
        return 1;
    else
        return 0;
}

```

### **Algorithm for PUSH and POP operations on Stack**

Let Stack[MAXSIZE] be an array to implement the stack. The variable top denotes the top of the stack.

#### **i) Algorithm for PUSH (inserting an item into the stack) operation:**

This algorithm adds or inserts an item at the top of the stack

1. [Check for stack overflow?]
  - if top=MAXSIZE-1 then
    - print "Stack Overflow" and Exit
  - else
    - Set top=top+1 [Increase top by 1]



*Set Stack[top]:= item [Inserts item in new top position]*

2. Exit

## ii) **Algorithm for POP (removing an item from the stack) operation**

This algorithm deletes the top element of the stack and assign it to a variable *item*

```

1. [Check for the stack Underflow]
   If top<0 then
       Print "Stack Underflow" and Exit
   else
       [Remove the top element]
       Set item=Stack [top]
       [Decrement top by 1]
       Set top=top-1
       Return the deleted item from the stack
2. Exit

```

## **The PUSH and POP functions**

The C function for **push** operation

```

void push(st *s, int element)
{
    if(isfull(s))    /* Checking Overflow condition */
        printf("\n \n The stack is overflow: Stack Full!!\n");
    else
        s->items[++(s->top)]=element; /* First increase top by 1 and store element at top position*/
}

```

## **OR**

Alternatively we can define the push function as give below:

```

void push()
{
    int item;
    if(top == MAXSIZE - 1)    //Checking stack overflow
        printf("\n The Stack Is Full");
    else
    {
        printf("Enter the element to be inserted");
        scanf("%d",&item); //reading an item
        top= top+1;          //increase top by 1
        stack[top] = item;    //storing the item at the top of
                               the stack
    }
}

```

## **The C function for POP operation**

```

void pop(stack *s)
{
    if(isempty(s))
        printf("\n\nstack Underflow: Empty Stack!!!");
    else
        printf("\nthe deleted item is %d:\t",s->items[s->top--]);/*deletes top element and
        decrease top by 1 */
}

```

## **OR**

Alternatively we can define the push function as give below:

```

void pop()
{
    int item;
    if(top < 0) //Checking Stack Underflow
        printf("The stack is Empty");
    else
    {
        item = stack[top]; //Storing top element to item variable
        top = top-1; //Decrease top by 1
        printf("The popped item is=%d", item); //Displaying the deleted item
    }
}

```

**The following complete program illustrates the implementation of stack with operations:**

**Program: 1**

```

/* Array Implementation of Stack */
#include<stdio.h>
#include<conio.h>
#define MAX 10
struct stack
{
    int items[MAX]; //Declaring an array to store items
    int top; //Top of a stack
};
typedef struct stack st;
void create_empty_stack(st *s); //function prototype
void push(st *s, int);
void pop(st *s);
void display(st *s);
//Main Function
void main()
{
    int element, choice;
    st *s;
    int flag=1;
    clrscr();
    create_empty_stack(s); /* s->top=-1; indicates empty stack */
    do
    {
        printf("\n\n Enter your choice");
        printf(" \n\n\t 1:Push the elements");
        printf(" \n\n\t 2: To display the elements");
        printf(" \n\n\t 3: Pop the element");
        printf(" \n\n\t 4: Exit");
        printf("\n\n\n Enter of your choice:\t");
        scanf("%d",&choice);
        switch(choice)
        {
            case 1:
                printf("\n Enter the number:");
                scanf("%d", &element); /*Read an element from keyboard*/
                push(s,element);
                break;
            case 2:
                display(s);

```

```

        break;
    case 3: clrscr();
        pop(s);
        break;
    case 4:
        flag=0;
        break;
    default:
        printf("\n Invalid Choice");
    }
} while(flag);
getch();
}
/*Function to create an empty stack*/
void create_empty_stack(st *s)
{
    s->top=-1;
}
/*Function to check whether the stack is empty or not */
int isempty(st *s)
{
    if(s->top==-1)
        return 1;
    else
        return 0;
}
/*function to check whether the stack is full or not*/
int isfull(st *s)
{
    if(s->top==MAX-1)
        return 1;
    else
        return 0;
}
/* push() function definition */
void push(st *s, int element)
{
    if(isfull(s)) /* Checking Overflow condition */
        printf("\n \nThe stack is overflow: Stack Full!!\n");
    else
        s->items[++(s->top)]=element;
}
/* Function for displaying elements of a stack */
void display(st *s)
{
    int i;
    if(isempty(s))
        printf("\nThe Stack does not contain any Elements");
    else
    {
        printf("\nThe elements in the stack is/are:\n");
        for(i=s->top; i>=0; i--)
            printf("%d\n", s->items[i]);
    }
}
}

```

```
/* the POP function definition*/
```

```
void pop(st *s)
{
    if(isempty(s))
        printf("\n\nstack Underflow: Empty Stack!!!");
    else
        printf("\n\nthe deleted item is %d:\t",s->items[s->top--]);
}
```

### **Lab work No:4**

```
/*implementation of stack by 2nd ways*/
```

```
#include<stdio.h>
#include<conio.h>
#define MAX 100
struct stack
{
    int item[MAX];
    int tos;
};
typedef struct stack st;
void push(st*, int);
int pop(st*);
void display(st*);
void main()
{
    int dta, ch, x;
    st *s;
    clrscr();
    s->tos=-1;
    printf("\n*****menu for program*****:\n");
    printf("1:push\n2:pop\n3:display\n4:exit\n");
    do
    {
        printf("\nEnter your choice\n");
        scanf("%d",&ch);
        switch(ch)
        {
            case 1:
                printf("Enter data to be inserted\n");
                scanf("%d",&dta);
                push(s,dta);
                break;
            case 2:
                x=pop(s);
                printf("\npopped item is:");
                printf("%d\n",x);
                break;
            case 3:
                display(s);
                break;
            default:
                exit(1);
        }
    } while(ch<4);
    getch();
}
```

\*\*\*\*\*push function\*\*\*\*\*

```

void push(st *s,int d)
{
    if(s->tos==MAX-1)
    {
        printf("Stack is full\n");
    }
    else
    {
        ++s->tos;
        s->item[s->tos]=d;
    }
}

/*****pop function*****/
int pop(st *s)
{
    int itm;
    if(s->tos==1)
    {
        printf("Stack is empty\n");
        return(0);
    }
    else
    {
        itm=s->item[s->tos];
        s->tos--;
        return(itm);
    }
}

/*****display function*****/
void display(st *s)
{
    int i;
    if(s->tos==1)
        printf("There is no data item to display\n");
    else
    {
        for(i=s->tos; i>=0; i--)
        {
            printf("%d\t", s->item[i]);
        }
    }
}

```

### **Infix, Prefix and Postfix Notation**

One of the applications of the stack is to evaluate the expression. We can represent the expression following three types of notation:

- Infix
- Prefix
- Postfix

- ❖ **Infix expression**: It is an ordinary mathematical notation of expression where operator is written in between the operands. Example:  $A+B$ . Here '+' is an *operator* and  $A$  and  $B$  are called *operands*
- ❖ **Prefix notation**: In prefix notation the operator precedes the two operands. That is the operator is written before the operands. It is also called polish notation. Example:  $+AB$
- ❖ **Postfix notation**: In postfix notation the operators are written after the operands so it is called the postfix notation (post mean after). In this notation the operator follows the two operands. Example:  $AB+$

**Examples:** $A + B$  (Infix) $+ AB$  (Prefix) $AB +$  (Postfix)

- Both prefix and postfix are parenthesis free expressions. For example
 

$(A + B) * C$	Infix form
$* + A B C$	Prefix form
$A B + C *$	Postfix form

Infix	Postfix	Prefix
$A+B$	$AB+$	$+AB$
$A+B-C$	$AB+C-$	$-+ABC$
$(A+B)*(C-D)$	$AB+CD-*$	$*+AB-CD$

**Converting an Infix Expression to Postfix**

**First convert the sub-expression to postfix that is to be evaluated first and repeat this process.** You substitute intermediate postfix sub-expression by any variable whenever necessary that makes it easy to convert.

- ❖ Remember, to convert an infix expression to its postfix equivalent, we first convert the innermost parenthesis to postfix, resulting as a new operand
- In this fashion parenthesis can be successively eliminated until the entire expression is converted
- ❖ The last pair of parenthesis to be opened within a group of parenthesis encloses the first expression within the group to be transformed
- This last in, first-out behavior suggests the use of a stack

**Precedence rule:**

While converting infix to postfix you have to consider the **precedence rule**, and the precedence rules are as follows

1. Exponentiation ( the expression  $A^B$  is A raised to the B power, so that  $3^2=9$ )
2. Multiplication/Division
3. Addition/Subtraction

When un-parenthesized operators of the same precedence are scanned, the order is assumed to be left to right except in the case of exponentiation, where the order is assumed to be from right to left.

- $A+B+C$  means  $(A+B)+C$
- $A^B^C$  means  $A^{(B^C)}$

By using parenthesis we can override the default precedence.

Consider an example that illustrate the converting of infix to postfix expression,  $A + (B * C)$ .

Use the following **rule** to convert it in postfix:

1. Parenthesis for emphasis
2. Convert the multiplication
3. Convert the addition
4. Post-fix form

#### Illustration:

$A + (B * C)$ . Infix form  
 $A + (B * C)$  Parenthesis for emphasis  
 $A + (BC^*)$  Convert the multiplication  
 $A (BC^*) +$  Convert the addition  
 $ABC^*+$  Post-fix form

#### Consider an example:

$(A + B) * ((C - D) + E) / F$  Infix form  
 $(AB+) * ((C - D) + E) / F$   
 $(AB+) * ((CD-) + E) / F$   
 $(AB+) * (CD-E+) / F$   
 $(AB+CD-E+*) / F$   
 $AB+CD-E+*F/$  Postfix form

#### Examples

<i>Infix</i>	<i>Postfix</i>
$A + B$	$AB +$
$A + B - C$	$AB + C -$
$(A + B) * (C - D)$	$AB + CD - *$
$A * B * C - D + E / F / (G + H)$	$AB * C * D - EF / GH + / +$
$((A + B) * C - (D - E)) * (F + G)$	$AB + C * DE - - FG + *$
$A - B / (C * D * E)$	$ABCDE * * / -$

**exercise:** Convert the infix expression listed in the above table into postfix notation and verify yourself.

**Algorithm to convert infix to postfix notation**

Let here two stacks opstack and poststack are used and otos & ptos represents the opstack top and poststack top respectively.

1. Scan one character at a time of an infix expression from left to right
2. opstack=the empty stack
3. Repeat till there is data in infix expression
  - 3.1 if scanned character is '(' then push it to opstack
  - 3.2 if scanned character is operand then push it to poststack
  - 3.3 if scanned character is operator then
    - if(opstack!= -1)
    - while(precedence (opstack[otos])>precedence(scan character)) then
      - pop and push it into poststack
    - otherwise
      - push into opstack
  - 3.4 if scanned character is ')' then
    - pop and push into poststack until '(' is not found and ignore both symbols
4. pop and push into poststack until opstack is not empty.
5. return

**Trace of Conversion Algorithm**

The following tracing of the algorithm illustrates the algorithm. Consider an infix expression

$((A-(B+C))*D)\$(E+F)$

Scan character	Poststack	opstack
(	.....	(
(	.....	((
A	A	((
-	A	(( -
(	A	(( -(
B	AB	(( -(
+	AB	(( -( +
C	ABC	(( -( +
)	ABC+	(( -
)	ABC+-	(
*	ABC+-	(*
D	ABC+-D	(*
)	ABC+-D*	.....
\$	ABC+-D*	\$
(	ABC+-D*	\$(
E	ABC+-D*E	\$(
+	ABC+-D*E	\$(+
F	ABC+-D*EF	\$(+
)	ABC+-D*EF+	\$
.....	ABC+-D*EF+\$ (postfix)	.....



**Converting an Infix expression to Prefix expression**

The precedence rule for converting from an expression from infix to prefix are identical.

Only changes from postfix conversion is that the operator is placed before the operands rather than after them. The prefix of

$A+B-C$  is  $-+ABC$ .

$A+B-C$  (infix)

$=(+AB)-C$

$=-+ABC$  (prefix)

*Example Consider an example:*

$A \ \$ \ B \ * \ C \ - \ D \ + \ E \ / \ F \ / \ (G \ + \ H)$     infix form

$= A \ \$ \ B \ * \ C \ - \ D \ + \ E \ / \ F \ / \ (+GH)$

$= \$AB * C - D + E / F / (+GH)$

$= * \$ABC - D + E / F / (+GH)$

$= * \$ABC - D + (/EF) / (+GH)$

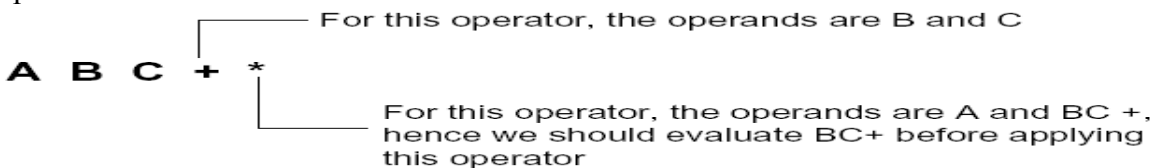
$= * \$ABC - D + //EF + GH$

$= (- * \$ABCD) + (/EF + GH)$

$= + - * \$ABCD //EF + GH$     which is in prefix form.

**Evaluating the Postfix expression**

Each operator in a postfix expression refers to the previous two operands in the expression.



To evaluate the postfix expression we use the following procedure:

Each time we read an operand we push it onto a stack. When we reach an operator, its operand s will be the top two elements on the stack. We can then pop these two elements perform the indicated operation on them and push the result on the stack so that it will be available for use as an operand of the next operator.

**Consider an example**

$3 \ 4 \ 5 \ * \ +$

$= 3 \ 20 \ +$

$= 23$  (answer)

**Evaluating the given postfix expression:**

$6 \ 2 \ 3 \ + \ - \ 3 \ 8 \ 2 \ / \ + \ * \ 2 \ \$ \ 3 \ +$

$= 6 \ 5 \ - \ 3 \ 8 \ 2 \ / \ + \ * \ 2 \ \$ \ 3 \ +$

$= 1 \ 3 \ 8 \ 2 \ / \ + \ * \ 2 \ \$ \ 3 \ +$

$= 1 \ 3 \ 4 \ + \ * \ 2 \ \$ \ 3 \ +$

$= 1 \ 7 \ * \ 2 \ \$ \ 3 \ +$

$= 7 \ 2 \ \$ \ 3 \ +$

$= 49 \ 3 \ +$

$= 52$

**Algorithm to evaluate the postfix expression**

Here we use only one stack called vstack(value stack).

1. Scan one character at a time from left to right of given postfix expression
  - 1.1 if scanned symbol is operand then  
read its corresponding value and push it into vstack
  - 1.2 if scanned symbol is operator then
    - pop and place into op2
    - op and place into op1
    - compute result according to given operator and push result into vstack
2. pop and display which is required value of the given postfix expression
3. return

**Trace of Evaluation:**

Consider an example to evaluate the postfix expression tracing the algorithm

**ABC+\*CBA-+\***

**123+\*321-+\***

Scanned character	value	Op2	Op1	Result	vstack
A	1	.....	.....	.....	1
B	2	.....	.....	.....	1 2
C	3	.....	.....	.....	1 2 3
+	.....	3	2	5	1 5
*	.....	5	1	5	5
C	3	.....	.....	.....	5 3
B	2	...	.....	.....	5 3 2
A	1	.....	.....	.....	5 3 2 1
-	.....	1	2	1	5 3 1
+	.....	1	3	4	5 4
*	.....	4	5	20	20

Its final value is 20.

**Evaluating the Prefix Expression**

To evaluate the prefix expression we use two stacks and some time it is called two stack algorithms. One stack is used to store operators and another is used to store the operands. Consider an example for this

+ 5 \*3 2 prefix expression

= +5 6

=11

**Illustration:** Evaluate the given prefix expression

/ + 5 3 – 4 2      prefix equivalent to (5+3)/(4-2) infix notation

= / 8 – 4 2

= / 8 2

= 4

```
/*program for evaluating postfix expression*/
```

```
#include<stdio.h>
#include<conio.h>
#include<math.h>
#include<string.h>
void push(int);
int pop();
int vstack[100];
int tos=-1;
void main()
{
    int i,res,l,op1,op2,value[100];
    char postfix[100],ch;
    clrscr();
    printf("Enter a valid postfix\n");
    gets(postfix);
    l=strlen(postfix);
    for(i=0;i<=l-1;i++)
    {
        if(isalpha(postfix[i]))
        {
            printf("Enter value of %c",postfix[i]);
            scanf("%d",&value[i]);
            push(value[i]);
        }
        else
        {
            ch=postfix[i];
            op2=pop();
            op1=pop();
            switch(ch)
            {
                case '+':
                    push(op1+op2);
                    break;
                case '-':
                    push(op1-op2);
                    break;
                case '*':
                    push(op1*op2);
                    break;
                case '/':
                    push(op1/op2);
                    break;
                case '$':
                    push(pow(op1,op2));
                    break;
                case '%':
                    push(op1%op2);
                    break;
            }
        }
    }
    printf("The result is:");
    res=pop();
    printf("%d", res);
}
```

```

        getch();
    }
    /*******insertion function*****/
    void push(int val)
    {
        vstack[++tos]=val;
    }
    /*******deletion function*****/
    int pop()
    {
        int n;
        n=vstack[tos--];
        return(n);
    }

```

**/\*program to convert infix to postfix expression\*/**

```

#include<stdio.h>
#include<conio.h>
#include<math.h>
#include<string.h>
int precedence(char);
void main()
{
    int i,otos=-1,ptos=-1,l,11;
    char infix[100],poststack[100],opstack[100];
    printf("Enter a valid infix\n");
    gets(infix);
    l=strlen(infix);
    11=1;
    for(i=0;i<=l-1;i++)
    {
        if(infix[i]=='(')
        {
            opstack[++otos]=infix[i];
            11++;
        }
        else if(isalpha(infix[i]))
        {
            poststack[++ptos]=infix[i];
        }
        else if (infix[i]==')')
        {
            11++;
            while(opstack[otos]!='(')
            {
                poststack[++ptos]=opstack[otos];
                otos--;
            }
            otos--;
        }
        else //operators
        {
            if(precedency(opstack[otos])>precedency(infix[i]))
            {
                poststack[++ptos]=opstack[otos--];
            }
        }
    }
    poststack[++ptos]=opstack[otos];
}

```

```

        opstack[++otos]=infix[i];
    }
    opstack[++otos]=infix[i];
}

}
while(otos!=-1)
{
    poststack[++ptos]=opstack[otos];
    otos--;
}
/*****for displaying*****/
for(i=0;i<11;i++)
{
    printf("%c",poststack[i]);
}
getch();
}
/*****precedency function*****/
int precedency(char ch)
{
    switch(ch)
    {
        case '$':
            return(4);
            // break;
        case '*':
        case '/':
            return(3);
            // break;
        case '+':
        case '-':
            return(2);
            // break;
        default:
            return(1);
    }
}

```

## **Recursion:**

Recursion is a process by which a function calls itself repeatedly, until some specified condition has been satisfied. The process is used for repetitive computations in which each action is stated in terms of a previous result.

In order to solve a problem recursively, two conditions must be satisfied. First, the problem must be written in a recursive form, and second, the problem statement must include a stopping condition.

**Example:**

```

/*calculation of the factorial of an integer number using recursive function*/
#include<stdio.h>
#include<conio.h>
void main()
{
    int n;
    long int facto;
    long int factorial(int n);
    printf("Enter value of n:");
    scanf("%d",&n);
    facto=factorial(n);
    printf("%d! = %ld",n,facto);
    getch();
}
long int factorial(int n)
{
    if(n == 0)
        return 1;
    else
        return n * factorial(n-1);
}

```

**Let's trace the evaluation of factorial(5):**

```

Factorial(5)=
5*Factorial(4)=
5*(4*Factorial(3))=
5*(4*(3*Factorial(2)))=
5*(4*(3*(2*Factorial(1))))=
5*(4*(3*(2*(1*Factorial(0)))))=
5*(4*(3*(2*(1*1))))=
5*(4*(3*(2*1)))=
5*(4*(3*2))=
5*(4*6)=
5*24=
120

```

**Example:**

```

/*calculation of the factorial of an integer number without using recursive function*/
#include<stdio.h>
#include<conio.h>
void main()
{
    int n;
    long int facto;
    long int factorial(int n);
    printf("Enter value of n:");
    scanf("%d",&n);
    facto=factorial(n);
    printf("%d! = %ld",n,facto);
    getch();
}

```

```

long int factorial(int n)
{
    long int facto=1;
    int i;
    if(n==0)
        return 1;
    else {
        for(i=1;i<=n;i++)
            facto=facto*i;
        return facto;
    }
}

```

**/\* Program to generate Fibonacci series up to n terms using recursive function\*/**

```

#include<stdio.h>
#include<conio.h>
void main()
{
    int n,i;
    int fibo(int);
    printf("Enter n:");
    scanf("%d",&n);
    printf("Fibonacci numbers up to %d terms:\n",n);
    for(i=1;i<=n;i++)
        printf("%d\n",fibo(i));
    getch();
}

```

```

int fibo(int k)
{
    if(k == 1 || k == 2)
        return 1;
    else
        return fibo(k-1)+fibo(k-2);
}

```

**/\* Program to find sum of first n natural numbers using recursion\*/**

```

#include<stdio.h>
#include<conio.h>
void main()
{
    int n;
    int sum_natural(int );
    printf("n = ");
    scanf("%d",&n);
    printf("Sum of first %d natural numbers = %d",n,sum_natural(n));
    getch();
}
int sum_natural(int n)
{
    if(n == 1)
        return 1;
    else
        return n + sum_natural(n-1);
}

```

## **Tower of Hanoi problem:**

### **Initial state:**

- There are three poles named as origin, intermediate and destination.
- **n** number of different-sized disks having hole at the center is stacked around the origin pole in decreasing order.
- The disks are numbered as 1, 2, 3, 4, .....n.

### **Objective:**

- Transfer all disks from origin pole to destination pole using intermediate pole for temporary storage.

### **Conditions:**

- Move only one disk at a time.
- Each disk must always be placed around one of the pole.
- Never place larger disk on top of smaller disk.

**Algorithm:** - To move a tower of  $n$  disks from *source* to *dest* (where  $n$  is positive integer):

1. If  $n == 1$ :
  - 1.1. Move a single disk from *source* to *dest*.
2. If  $n > 1$ :
  - 2.1. Let *temp* be the remaining pole other than *source* and *dest*.
  - 2.2. Move a tower of  $(n - 1)$  disks from *source* to *temp*.
  - 2.3. Move a single disk from *source* to *dest*.
  - 2.4. Move a tower of  $(n - 1)$  disks from *temp* to *dest*.
3. Terminate.

### **Example: Recursive solution of tower of Hanoi:**

```
#include <stdio.h>
#include <conio.h>
void TOH(int, char, char, char); //Function prototype
void main()
{
    int n;
    printf("Enter number of disks");
    scanf("%d",&n);
    TOH(n,'O','D','I');
    getch();
}

void TOH(int n, char A, char B, char C)
{
    if(n>0)
    {
        TOH(n-1, A, C, B);
        Printf("Move disk %d from %c to %c\n", n, A, B);
        TOH(n-1, C, B, A);
    }
}
```



```

/* Program to find multiplication of first n natural numbers using
recursion*/
#include<stdio.h>
#include<conio.h>
void main()
{
    int n;
    int mul_natural(int );
    printf("n = ");
    scanf("%d",&n);
    printf("Product of first %d natural numbers = %d", n, mul_natural(n));
    getch();
}
int mul_natural(int n)
{
    if(n == 1)
        return 1;
    else
        return (n * mul_natural(n-1));
}

```

**Advantages of Recursion:**

- The code may be much easier to write.
- To solve some problems which are naturally recursive such as tower of Hanoi.

**Disadvantages of Recursion:**

- Recursive functions are generally slower than non-recursive functions.
- May require a lot of memory to hold intermediate results on the system stack.
- It is difficult to think recursively so one must be very careful when writing recursive functions.

**Unit 4****Queues**

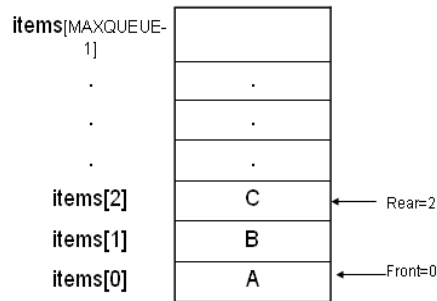
- Concept and definition
- Queue as ADT
- Implementation of insert and delete operation of
  - Linear queue
  - Circular queue
- Concept of priority queue

**What is a queue?**

> A **Queue** is an ordered collection of items from which items may be deleted at one end (called the **front** of the queue) and into which items may be inserted at the other end (the **rear** of the queue).

> The first element inserted into the queue is the first element to be removed. For this reason a queue is sometimes called a **fifo** (first-in first-out) list as opposed to the stack, which is a **lifo** (last-in first-out).

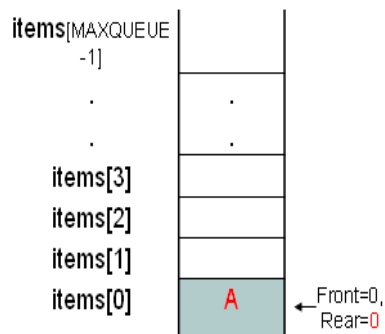
### Example:



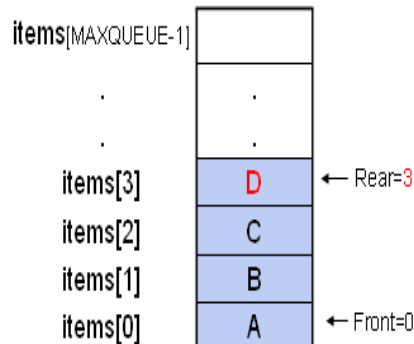
### Operations on queue:

- **MakeEmpty(q):** To make q as an empty queue
- **Enqueue(q, x):** To insert an item x at the rear of the queue, this is also called by names add, insert.
- **Dequeue(q):** To delete an item from the front of the queue q. this is also known as Delete, Remove.
- **IsFull(q):** To check whether the queue q is full.
- **IsEmpty(q):** To check whether the queue q is empty
- **Traverse (q):** To read entire queue that is display the content of the queue.

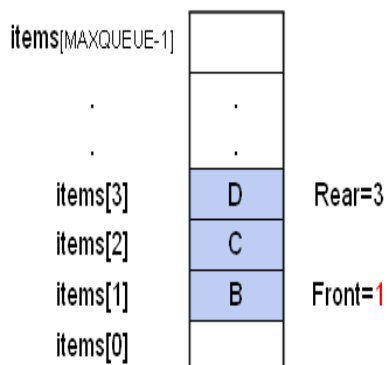
### Enqueue(A):



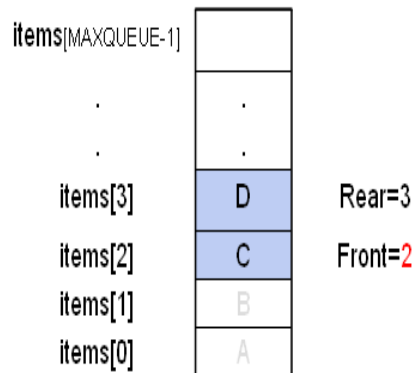
### Enqueue(B,C,D):



### Dequeue(A):

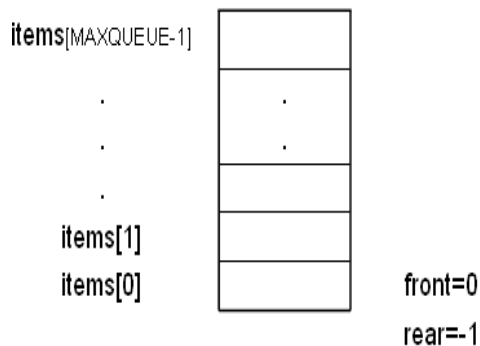


### Dequeue(B):



**Initialization of queue:**

- The queue is initialized by having the *rear* set to  $-1$ , and *front* set to  $0$ . Let us assume that maximum number of the element we have in a queue is MAXQUEUE elements as shown below.

**Applications of queue:**

- Task waiting for the printing
- Time sharing system for use of CPU
- For access to disk storage
- Task scheduling in operating system

**The Queue as a ADT:**

A queue  $q$  of type  $T$  is a finite sequence of elements with the operations

- MakeEmpty( $q$ )***: To make  $q$  as an empty queue
- IsEmpty( $q$ )***: To check whether the queue  $q$  is empty. Return true if  $q$  is empty, return false otherwise.
- IsFull( $q$ )***: To check whether the queue  $q$  is full. Return true in  $q$  is full, return false otherwise.
- Enqueue( $q, x$ )***: To insert an item  $x$  at the rear of the queue, if and only if  $q$  is not full.
- Dequeue( $q$ )***: To delete an item from the front of the queue  $q$ . if and only if  $q$  is not empty.
- Traverse ( $q$ )***: To read entire queue that is display the content of the queue.

**Implementation of queue:**

There are two techniques for implementing the queue:

- Array implementation of queue(static memory allocation)
- Linked list implementation of queue(dynamic memory allocation)

**Array implementation of queue:**

In array implementation of queue, an array is used to store the data elements. Array implementation is also further classified into two types

✓ ***Linear array implementation:***

A linear array with two indices always increasing that is rear and front. Linear array implementation is also called linear queue

- ✓ **Circular array implementation:**  
This is also called circular queue.

### **Linear queue:**

#### **Algorithm for insertion (or Enqueue ) and deletion (Dequeue) in queue:**

##### **Algorithm for insertion an item in queue:**

1. Initialize  $front=0$  and  $rear=-1$   
if  $rear \geq MAXSIZE-1$   
print "queue overflow" and return  
else  
set  $rear=rear+1$   
 $queue[rear]=item$
2. end

##### **Algorithm to delete an element from the queue:**

1. if  $rear < front$   
print "queue is empty" and return  
else  
 $item=queue[front++]$
2. end

##### **Declaration of a Queue:**

```
# define MAXQUEUE 50 /* size of the queue items*/
struct queue
{
    int front;
    int rear;
    int items[MAXQUEUE];
};
typedef struct queue qt;
```

##### **Defining the operations of linear queue:**

- **The MakeEmpty function:**

```
void makeEmpty(qt *q)
{
    q->rear=-1;
    q->front=0;
}
```

- **The IsEmpty function:**

```
int IsEmpty(qt *q)
{
    if(q->rear < q->front)
        return 1;
    else
        return 0;
}
```

- **The Isfull function:**

```
int IsFull(qt *q)
```

```

{
    if(q->rear==MAXQUEUEZIZE-1)
        return 1;
    else
        return 0;
}

```

• **The Enqueue function:**

```

void Enqueue(qt *q, int newitem)
{
    if(IsFull(q))
    {
        printf("queue is full");
        exit(1);
    }
    else
    {
        q->rear++;
        q->items[q->rear]=newitem;
    }
}

```

• **The Dequeue function:**

```

int Dequeue(qt *q)
{
    if(IsEmpty(q))
    {
        printf("queue is Empty");
        exit(1);
    }
    else
    {
        return(q->items[q->front]);
        q->front++;
    }
}

```

**Problems with Linear queue implementation:**

- Both rear and front indices are increased but never decreased.
- As items are removed from the queue, the storage space at the beginning of the array is discarded and never used again. Wastage of the space is the main problem with linear queue which is illustrated by the following example.

		11	22	33	44	55
0	1	2	3	4	5	6
		f				r

front=2, rear=6

This queue is considered full, even though the space at beginning is vacant.

***/\*Array implementation of linear queue\*/***

```

#include<stdio.h>
#include<conio.h>
#define SIZE 20
struct queue
{
    int item[SIZE];
    int rear;
    int front;
};
typedef struct queue qu;
void insert(qu*);
void delet(qu*);
void display(qu*);
void main()
{
    int ch;
    qu *q;
    q->rear=-1;
    q->front=0;
    clrscr();
    printf("Menu for program:\n");
    printf("1:insert\n2:delete\n3:display\n4:exit\n");
    do
    {
        printf("Enter your choice\n");
        scanf("%d",&ch);
        switch(ch)
        {
            case 1:
                insert(q);
                break;
            case 2:
                delet(q);
                break;
            case 3:
                display(q);
                break;
            case 4:
                exit(1);
                break;
            default:
                printf("Your choice is wrong\n");
        }
    } while(ch<5);
    getch();
}
/*****insert function*****/
void insert(qu *q)
{
    int d;
    printf("Enter data to be inserted\n");
    scanf("%d",&d);

```

```

    if(q->rear==SIZE-1)
    {
        printf("Queue is full\n");
    }
    else
    {
        q->rear++;
        q->item[q->rear]=d;
    }
}
/*****delete function*****/
void delet(qu *q)
{
    int d;
    if(q->rear<q->front)
    {
        printf("Queue is empty\n");
    }
    else
    {
        d=q->item[q->front];
        q->front++;
        printf("Deleted item is:");
        printf("%d\n",d);
    }
}
/*****display function*****/
void display(qu *q)
{
    int i;
    if(q->rear<q->front)
    {
        printf("Queue is empty\n");
    }
    else
    {
        for(i=q->front;i<=q->rear;i++)
        {
            printf("%d\t",q->item[i]);
        }
    }
}

```

### **Circular queue:**

A circular queue is one in which the insertion of a new element is done at very first location of the queue if the last location of the queue is full.

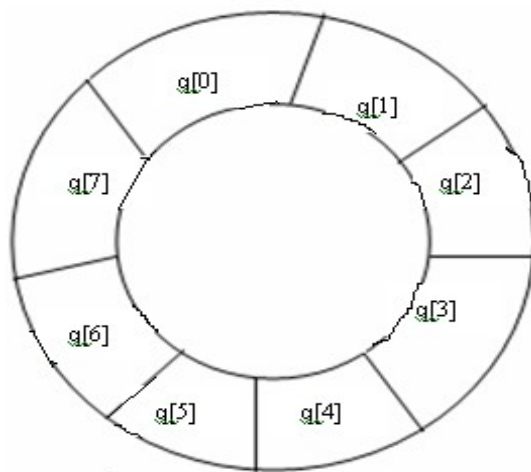


Fig:- Circular queue

- ◆ A circular queue overcomes the problem of unutilized space in linear queue implementation as array.
- ◆ In circular queue we sacrifice one element of the array thus to insert n elements in a circular queue we need an array of size n+1.(or we can insert one less than the size of the array in circular queue).

### **Initialization of Circular queue:**

rear=front=MAXSIZE-1

### **Algorithms for inserting an element in a circular queue:**

This algorithm is assume that rear and front are initially set to MAXSIZE-1.

1. if (front==(rear+1)%MAXSIZE)  
    print Queue is full and exit  
    else  
        rear=(rear+1)%MAXSIZE; [increment rear by 1]
2. cqueue[rear]=item;
3. end

### **Algorithms for deleting an element from a circular queue:**

This algorithm is assume that rear and front are initially set to MAXSIZE-1.

1. if (rear==front) [checking empty condition]  
    print Queue is empty and exit
2. front=(front+1)%MAXSIZE; [increment front by 1]
3. item=cqueue[front];
4. return item;
5. end.

### **Declaration of a Circular Queue:**

# define MAXSIZE 50 /\* size of the circular queue items\*/

struct cqueue

{

    int front;

    int rear;

    int items[MAXSIZE];

};

typedef struct cqueue cq;



**Operations of a circular queue:**◆ **The MakeEmpty function:**

```
void makeEmpty(cq *q)
{
    q->rear=MAXSIZE-1;
    q->front=MAXSIZE-1;
}
```

◆ **The IsEmpty function:**

```
int IsEmpty(cq *q)
{
    if(q->rear<q->front)
        return 1;
    else
        return 0;
}
```

◆ **The Isfull function:**

```
int IsFull(cq *q)
{
    if(q->front==(q->rear+1)%MAXDIZE)
        return 1;
    else
        return 0;
}
```

◆ **The Enqueue function:**

```
void Enqueue(cq *q, int newitem)
{
    if(IsFull(q))
    {
        printf("queue is full");
        exit(1);
    }
    else
    {
        q->rear=(q->rear+1)%MAXDIZE;
        q->items[q->rear]=newitem;
    }
}
```

◆ **The Dequeue function:**

```
int Dequeue(cq *q)
{
    if(IsEmpty(q))
    {
        printf("queue is Empty");
        exit(1);
    }
    else
    {
        q->front=(q->front+1)%MAXSIZE;
        return(q->items[q->front]);
    }
}
```

```

    }
}

/*implementation of circular queue with secrifying one cell */
#include<stdio.h>
#include<conio.h>
#define SIZE 20
struct cqueue
{
    int item[SIZE];
    int rear;
    int front;
};
typedef struct cqueue qu;
void insert(qu*);
void delet(qu*);
void display(qu*);
void main()
{
    int ch;
    qu *q;
    q->rear=SIZE-1;
    q->front=SIZE-1;
    clrscr();
    printf("Menu for program:\n");
    printf("1:insert\n2:delete\n3:display\n4:exit\n");
    do
    {
        printf("Enter youer choice\n");
        scanf("%d",&ch);
        switch(ch)
        {
            case 1:
                insert(q);
                break;
            case 2:
                delet(q);
                break;
            case 3:
                display(q);
                break;
            case 4:
                exit(1);
                break;
            default:
                printf("Your choice is wrong\n");
                break;
        }
    } while(ch<5);
    getch();
}

/*****insert function*****/
void insert(qu *q)
{
    int d;
    if((q->rear+1)%SIZE==q->front)
        printf("Queue is full\n");

```

```

else
{
    q->rear=(q->rear+1)%SIZE;
    printf("Enter data to be inserted\n");
    scanf("%d",&d);
    q->item[q->rear]=d;
}

}

/*****delete function*****/
void delet(qu *q)
{
    if(q->rear==q->front)
        printf("Queue is empty\n");
    else
    {
        q->front=(q->front+1)%SIZE;
        printf("Deleted item is:");
        printf("%d\n",q->item[q->front]);
    }
}

/*****display function*****/
void display(qu *q)
{
    int i;
    if(q->rear==q->front)
        printf("Queue is empty\n");
    else
    {
        printf("Items of queue are:\n");
        for(i=(q->front+1)%SIZE;i!=q->rear;i=(i+1)%SIZE)
        {
            printf("%d\t",q->item[i]);
        }
        printf("%d\t",q->item[q->rear]);
    }
}

```

***/\*implementation of circular queue without sacrificing one cell by using a count variable \*/***

```

#include<stdio.h>
#include<conio.h>
#define SIZE 20
struct cqueue
{
    int item[SIZE];
    int rear;
    int front;
};
int count=0;
typedef struct cqueue qu;
void insert(qu*);
void delet(qu*);
void display(qu*);
void main()
{

```

```

int ch;
qu *q;
q->rear=SIZE-1;
q->front=SIZE-1;
clrscr();
printf("Menu for program:\n");
printf("1:insert\n2:delete\n3:display\n4:exit\n");
do
{
    printf("Enter your choice\n");
    scanf("%d",&ch);
    switch(ch)
    {
        case 1:
            insert(q);
            break;
        case 2:
            delet(q);
            break;
        case 3:
            display(q);
            break;
        case 4:
            exit(1);
            break;
        default:
            printf("Your choice is wrong\n");
            break;
    }
} while(ch<5);
getch();
}

```

```

/*****insert function*****/

```

```

void insert(qu *q)
{
    int d;
    if(count==SIZE)
        printf("Queue is full\n");
    else
    {
        q->rear=(q->rear+1)%SIZE;
        printf("Enter data to be inserted\n");
        scanf("%d",&d);
        q->item[q->rear]=d;
        count++;
    }
}

```

```

/*****delete function*****/

```

```

void delet(qu *q)
{
    if(count==0)
        printf("Queue is empty\n");
    else
    {

```

```

        q->front=(q->front+1)%SIZE;
        printf("Deleted item is:");
        printf("%d\n",q->item[q->front]);
        count--;
    }
}

/*****display function*****/
void display(qu *q)
{
    int i;
    if(q->rear==q->front)
        printf("Queue is empty\n");
    else
    {
        printf("Items of queue are:\n");
        for(i=(q->front+1)%SIZE; i!=q->rear; i=(i+1)%SIZE)
        {
            printf("%d\t",q->item[i]);
        }
        printf("%d\t",q->item[q->rear]);
    }
}

```

### **Priority queue:**

A priority queue is a collection of elements such that each element has been assigned a priority and the order in which elements are deleted and processed comes from the following rules:

- ◆ An element of higher priority is processed before any element of lower priority.
- ◆ If two elements has same priority then they are processed according to the order in which they were added to the queue.

The best application of priority queue is observed in CPU scheduling.

- ✓ The jobs which have higher priority are processed first.
- ✓ If the priority of two jobs is same this jobs are processed according to their position in queue.
- ✓ A short job is given higher priority over the longer one.

### **Types of priority queues:**

#### ◆ **Ascending priority queue(min priority queue):**

An *ascending priority queue* is a collection of items into which items can be inserted arbitrarily but from which only the smallest item can be removed.

#### ◆ **Descending priority queue(max priority queue):**

An **descending priority queue** is a collection of items into which items can be inserted arbitrarily but from which only the largest item can be removed.

### **Priority QUEUE Operations:**

#### ◆ **Insertion :**

The insertion in Priority queues is **the same as** in non-priority queues.

#### ◆ **Deletion :**

Deletion requires a search for the element of highest priority and deletes the element with highest priority. The following methods can be used for deletion/removal from a given Priority Queue:

- ✓ An empty indicator replaces deleted elements.
- ✓ After each deletion elements can be moved up in the array decrementing the rear.
- ✓ The array in the queue can be maintained as an ordered circular array

### **Priority Queue Declaration:**

*Queue data type of Priority Queue is the same as the Non-priority Queue.*

```
#define MAXQUEUE 10 /* size of the queue items*/
```

```
struct pqueue
```

```
{
```

```
    int front;
```

```
    int rear;
```

```
    int items[MAXQUEUE];
```

```
};
```

```
struct pqueue *pq;
```

### **The priority queue ADT:**

A ascending priority queue of elements of type T is a finite sequence of elements of T together with the operations:

- ◆ **MakeEmpty(p):** Create an empty priority queue p
- ◆ **Empty(p):** Determine if the priority queue p is empty or not
- ◆ **Insert(p,x):** Add element x on the priority queue p
- ◆ **DeleteMin(p):** If the priority queue p is not empty, remove the minimum element of the queue and return it.
- ◆ **FindMin(p):** Retrieve the minimum element of the priority queue p.

### **Array implementation of priority queue:**

#### ◆ **Unordered array implementation:**

- ✓ To insert an item, insert it at the rear end of the queue.
- ✓ To delete an item, find the position of the minimum element and
  - ✗ Either mark it as deleted (lazy deletion) or
  - ✗ shift all elements past the deleted element by one position and then decrement rear.

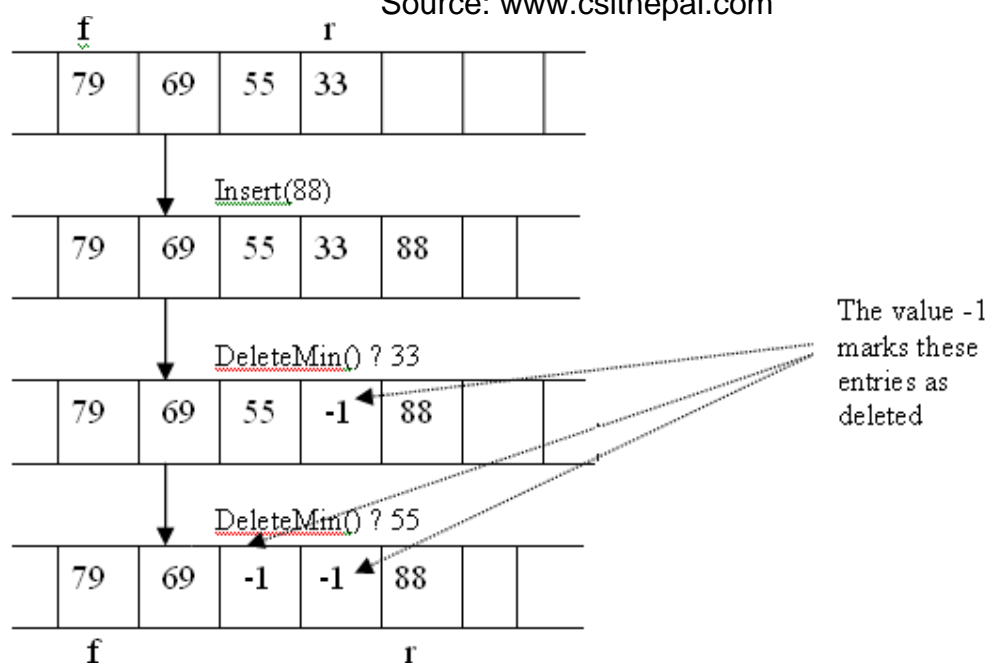


Fig Illustration of unordered array implementation

◆ **Ordered array implementation:**

- ✓ Set the front as the position of the smallest element and the rear as the position of the largest element.
- ✓ To insert an element, locate the proper position of the new element and shift preceding or succeeding elements by one position.
- ✓ To delete the minimum element, increment the front position.

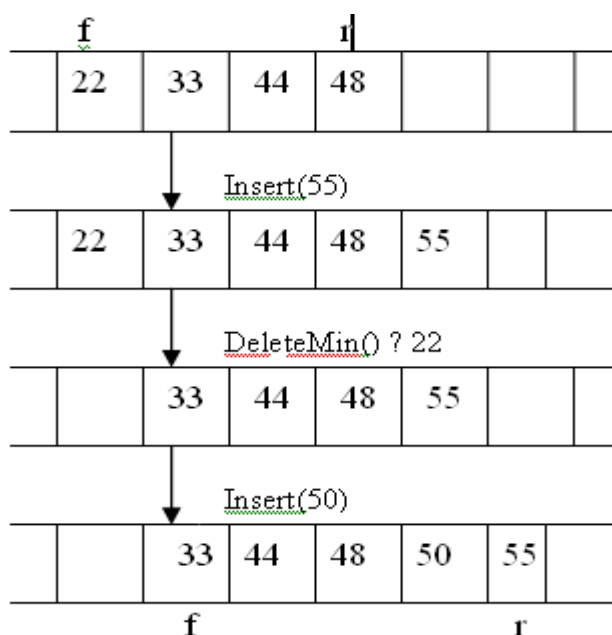


Fig Illustration of ordered array implementation

**Application of Priority queue:**

In a time-sharing computer system, a large number of tasks may be waiting for the CPU, some of these tasks have higher priority than others. The set of tasks waiting for the CPU forms a priority queue.

**/\*implementation of ascending priority queue \*/**

```
#include<stdio.h>
#include<conio.h>
#define SIZE 20
struct cqueue
{
    int item[SIZE];
    int rear;
    int front;
};
typedef struct queue pq;
void insert(pq*);
void delet(pq*);
void display(pq*);
void main()
{
    int ch;
    pq *q;
    q->rear=-1;
    q->front=0;
    clrscr();
    printf("Menu for program:\n");
    printf("1:insert\n2:delete\n3:display\n4:exit\n");
    do
    {
        printf("Enter your choice\n");
        scanf("%d",&ch);
        switch(ch)
        {
            case 1:
                insert(q);
                break;
            case 2:
                delet(q);
                break;
            case 3:
                display(q);
                break;
            case 4:
                exit(1);
                break;
            default:
                printf("Your choice is wrong\n");
                break;
        }
    }while(ch<5);
    getch();
}
```

/\*\*\*\*\*\*insert function\*\*\*\*\*\*/



```

void insert(pq *q)
{
    int d;
    if(q->rear==SIZE-1)
        printf("Queue is full\n");
    else
    {
        printf("Enter data to be inserted\n");
        scanf("%d",&d);
        q->rear++;
        q->item[q->rear]=d;
    }
}

/*****delete function*****/
void delet(pq *q)
{
    int i, temp=0, x;
    x=q->item[q->front];
    if(q->rear<q->front)
    {
        printf("Queue is empty\n");
        return 0;
    }
    else
    {
        for(i=q->front+1; i<q->rear; i++)
        {
            if(x>q->item[i])
            {
                temp=i;
                x=q->item[i];
            }
        }
        for(i=temp; i<q->rear-1; i++)
        {
            q->item[i]=q->item[i+1];
        }
        q->rear--;
        return x;
    }
}

/*****display function*****/
void display(pq *q)
{
    int i;
    if(q->rear < q->front)
        printf("Queue is empty\n");
    else
    {
        printf("Items of queue are:\n");
        for(i=q->front; i<=q->rear; i++)
        {
            printf("%d\t",q->item[i]);
        }
    }
}

```

```

    }
}

```

## Unit 4

### Linked List:

- Concept and definition
- Inserting and deleting nodes
- Linked implementation of a stack (PUSH / POP)
- Linked implementation of a queue (insert / delete)
- Circular linked list
  - ◆ Stack as a circular list (PUSH / POP)
  - ◆ Queue as a circular list (Insert / delete)
- Doubly linked list (insert / delete)

### Self referential structure:

It is sometimes desirable to include within a structure one member that is a pointer to the parent structure type. Hence, a structure which contains a reference to itself is called self-referential structure. In general terms, this can be expressed as:

```

struct node
{
    member 1;
    member 2;
    .....
    struct node *name;
};

```

**For example,**

```

struct node
{
    int info;
    struct node *next;
};

```

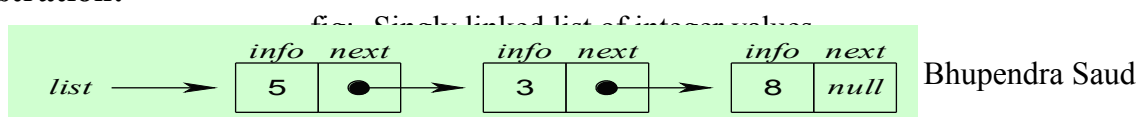
This is a structure of type node. The structure contains two members: a *info* integer member, and a pointer to a structure of the same type (i.e., a pointer to a structure of type node), called next. Therefore this is a **self-referential** structure.

### Linked List:

A linked list is a collection of nodes, where each node consists of two parts:

- ◆ **info:** the actual element to be stored in the list. It is also called data field.
- ◆ **link:** one or two links that points to next and previous node in the list. It is also called next or pointer field.

### **Illustration:**



- ◆ The nodes in a linked list are not stored contiguously in the memory
- ◆ You don't have to shift any element in the list.
- ◆ Memory for each node can be allocated dynamically whenever the need arises.
- ◆ The size of a linked list can grow or shrink dynamically

### **Operations on linked list:**

The basic operations to be performed on the linked list are as follows:

- ◆ **Creation:** This operation is used to create a linked list
- ◆ **Insertion:** This operation is used to insert a new node in a linked list in a specified position. A new node may be inserted
  - ✓ At the beginning of the linked list
  - ✓ At the end of the linked list
  - ✓ At the specified position in a linked list
- ◆ **Deletion:** The deletion operation is used to delete a node from the linked list. A node may be deleted from
  - ✓ The beginning of the linked list
  - ✓ the end of the linked list
  - ✓ the specified position in the linked list.
- ◆ **Traversing:** The list traversing is a process of going through all the nodes of the linked list from one end to the other end. The traversing may be either forward or backward.
- ◆ **Searching or find:** This operation is used to find an element in a linked list. In the desired element is found then we say operation is successful otherwise unsuccessful.
- ◆ **Concatenation:** It is the process of appending second list to the end of the first list.

### **Types of Linked List:**

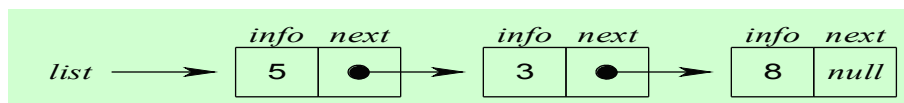
basically we can put linked list into the following four types:

- ◆ Singly linked list
- ◆ doubly linked list
- ◆ circular linked list
- ◆ circular doubly linked list

### **Singly linked list:**

A singly linked list is a dynamic data structure which may grow or shrink, and growing and shrinking depends on the operation made. In this type of linked list each node contains two fields one is info field which is used to store the data items and another is link field that is used to point the next node in the list. The last node has a NULL pointer.

The following example is a singly linked list that contains three elements 5, 3, 8.



### **Representation of singly linked list:**

We can create a structure for the singly linked list the each node has two members, one is **info** that is used to store the data items and another is **next** field that store the address of next node in the list.

We can define a node as follows:

```
struct Node
{
    int info;
    struct Node *next;
};
typedef struct Node NodeType;
NodeType *head; //head is a pointer type structure variable
```

This type of structure is called self-referential structure.

- ◆ *The NULL value of the next field of the linked list indicates the last node and we define macro for NULL and set it to 0 as below:*

```
#define NULL 0
```

### Creating a Node:

- ◆ To create a new node, we use the **malloc** function to dynamically allocate memory for the new node.
- ◆ After creating the node, we can store the new item in the node using a pointer to that node.

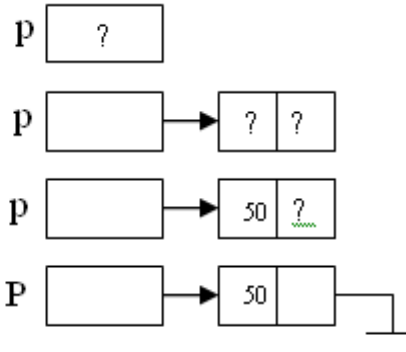
The following steps clearly shows the steps required to create a node and storing an item.

```
Nodetype *p;
```

```
P=(Nodetype*)malloc(sizeof(Nodetype));
```

```
p->info=50;
```

```
p->next=NULL;
```



*Note that p is not a node; instead it is a pointer to a node.*

### The getNode function:

we can define a function getNode() to allocate the memory for a node dynamically. It is user-defined function that return a pointer to the newly created node.

```
Nodetype *getNode()
{
    NodeType *p;
    p==(NodeType*)malloc(sizeof(NodeType));
    return(p);
}
```

### Creating the empty list:

```
void createEmptyList(NodeType *head)
{
    head=NULL;
}
```

### **Inserting Nodes:**

To insert an element or a node in a linked list, the following three things to be done:

- ◆ Allocating a node
- ◆ Assigning a data to info field of the node
- ◆ Adjusting a pointer and a new node may be inserted
  - ◆ At the beginning of the linked list
  - ◆ At the end of the linked list
  - ◆ At the specified position in a linked list

Insertion requires obtaining a new node and changing two links

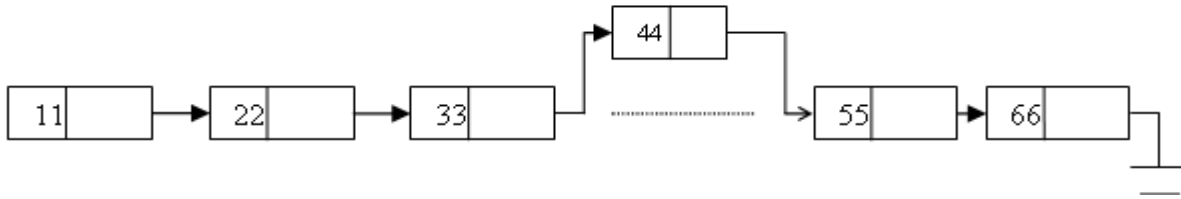


fig:- Inserting the new node with 44 between 33 and 55.

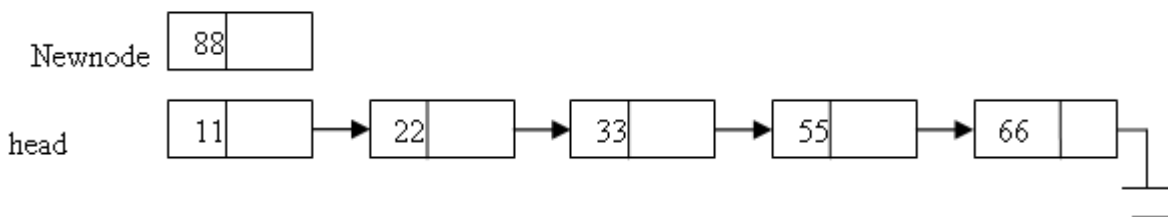
### **An algorithm to insert a node at the beginning of the singly linked list:**

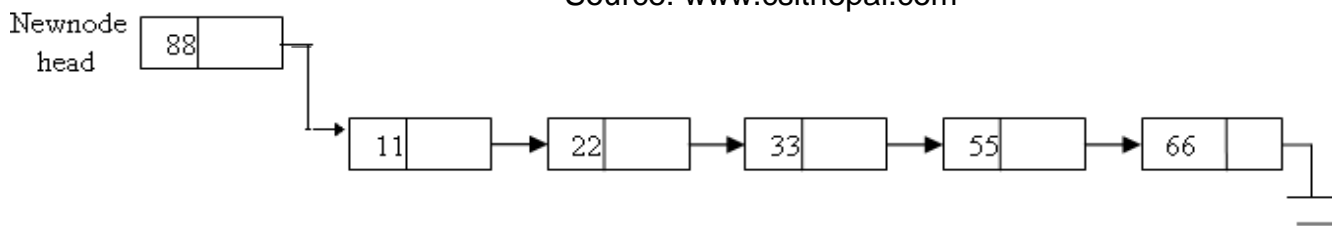
let \*head be the pointer to first node in the current list

1. Create a new node using malloc function  
*NewNode=(NodeType\*)malloc(sizeof(NodeType));*
2. Assign data to the info field of new node  
*NewNode->info=newItem;*
3. Set next of new node to head  
*NewNode->next=head;*
4. Set the head pointer to the new node  
*head=NewNode;*
5. End

### **The C function to insert a node at the beginning of the singly linked list:**

```
void InsertAtBeg(int newItem)
{
    NodeType *NewNode;
    NewNode=getNode();
    NewNode->info=newItem;
    NewNode->next=head;
    head=NewNode;
}
```





### **An algorithm to insert a node at the end of the singly linked list:**

let \*head be the pointer to first node in the current list

1. Create a new node using malloc function  
 $\text{NewNode} = (\text{NodeType}^*)\text{malloc}(\text{sizeof}(\text{NodeType}));$
2. Assign data to the info field of new node  
 $\text{NewNode} \rightarrow \text{info} = \text{newItem};$
3. Set next of new node to NULL  
 $\text{NewNode} \rightarrow \text{next} = \text{NULL};$
4. if (head == NULL) then  
     Set head = NewNode. and exit.
5. Set temp = head;
6. while (temp->next != NULL)  
     temp = temp->next; //increment temp
7. Set temp->next = NewNode;
8. End

### **The C function to insert a node at the end of the linked list:**

```
void InsertAtEnd(int newItem)
{
    NodeType *NewNode;
    NewNode=getNode();
    NewNode->info=newItem;
    NewNode->next=NULL;
    if(head==NULL)
    {
        head=NewNode;
    }
    else
    {
        temp=head;
        while(temp->next!=NULL)
        {
            temp=temp->next;
        }
        temp->next=NewNode;
    }
}
```

### **An algorithm to insert a node after the given node in singly linked list:**

let \*head be the pointer to first node in the current list and \*p be the pointer to the node after which we want to insert a new node.

1. Create a new node using malloc function  
 $\text{NewNode} = (\text{NodeType}^*)\text{malloc}(\text{sizeof}(\text{NodeType}));$
2. Assign data to the info field of new node  
 $\text{NewNode} \rightarrow \text{info} = \text{newItem};$
3. Set next of new node to next of p

*NewNode->next=p->next;*

4. Set next of p to NewNode  
    *p->next =NewNode..*
5. End

### **The C function to insert a node after the given node in singly linked list:**

```
void InsertAfterNode(NodeType *p int newItem)
{
    NodeType *NewNode;
    NewNode=getNode();
    NewNode->info=newItem;
    if(p==NULL)
    {
        printf("Void insertion");
        exit(1);
    }
    else
    {
        NewNode->next=p->next;
        p->next =NewNode..
    }
}
```

### **An algorithm to insert a node at the specified position in a singly linked list:**

let \*head be the pointer to first node in the current list

1. Create a new node using malloc function  
    *NewNode=(NodeType\*)malloc(sizeof(NodeType));*
2. Assign data to the info field of new node  
    *NewNode->info=newItem;*
3. Enter position of a node at which you want to insert a new node. Let this position is pos.
4. Set temp=head;
5. if (head ==NULL)then  
    printf("void insertion"); and exit(1).
6. for(i=1; i<pos-1; i++)  
    temp=temp->next;
7. Set *NewNode->next=temp->next;*  
    set temp->next =NewNode..
8. End

### **The C function to insert a node at the specified position in a singly linked list:**

```
void InsertAtPos(int newItem)
{
    NodeType *NewNode;
    int pos , i ;
    printf(" Enter position of a node at which you want to insert a new node");
    scanf("%d",&pos);
    if(head==NULL)
    {
        printf("void insertion");
        exit(1).
    }
    else
    {
        temp=head;
```

```

        for(i=1; i<pos-1; i++)
        {
            temp=temp->next;
        }
        NewNode=getNode();
        NewNode->info=newItem;
        NewNode->next=temp->next;
        temp->next =NewNode;
    }
}

```

### **Deleting Nodes:**

A node may be deleted:

- ◆ From the beginning of the linked list
- ◆ from the end of the linked list
- ◆ from the specified position in a linked list

### **Deleting first node of the linked list:**

#### **An algorithm to deleting the first node of the singly linked list:**

let \*head be the pointer to first node in the current list

1. If(head==NULL) then  
    print “Void deletion” and exit
2. Store the address of first node in a temporary variable **temp**.  
    temp=head;
3. Set head to next of head.  
    head=head->next;
4. Free the memory reserved by temp variable.  
    free(temp);
5. End

#### **The C function to deleting the first node of the singly linked list:**

```

void deleteBeg()
{
    NodeType *temp;
    if(head==NULL)
    {
        printf("Empty list");
        exit(1);
    }
    else
    {
        temp=head;
        printf("Deleted item is %d", head->info);
        head=head->next;
        free(temp);
    }
}

```

### **Deleting the last node of the linked list:**

#### **An algorithm to deleting the last node of the singly linked list:**

let \*head be the pointer to first node in the current list



1. If(head==NULL) then //if list is empty  
    print "Void deletion" and exit
2. else if(head->next==NULL) then //if list has only one node  
    Set temp=head;  
    print deleted item as,  
    printf("%d",head->info);  
    head=NULL;  
    free(temp);
3. else  
    set temp=head;  
    **while**(temp->next->next!=NULL)  
        set temp=temp->next;  
    End of **while**  
    **free**(temp->next);  
    Set temp->next=NULL;
4. End

### **The C function to deleting the last node of the singly linked list:**

let \*head be the pointer to first node in the current list

```
void deleteEnd()
{
    NodeType *temp;
    if(head==NULL)
    {
        printf("Empty list");
        return;
    }
    else if(head->next==NULL)
    {
        temp=head;
        head=NULL;
        printf("Deleted item is %d", temp->info);
        free(temp);
    }
    else
    {
        temp=head;
        while(temp->next->next!=NULL)
        {
            temp=temp->next;
        }
        printf("deleted item is %d", temp->next->info);
        free(temp->next);
        temp->next=NULL;
    }
}
```

### **An algorithm to delete a node after the given node in singly linked list:**

let \*head be the pointer to first node in the current list and \*p be the pointer to the node after which we want to delete a new node.

1. **if**(p==NULL or p->next==NULL) then  
    print "deletion not possible and exit"
2. set q=p->next
3. Set p->next=q->next;
4. **free**(q)

5. End

### **The C function to delete a node after the given node in singly linked list:**

let \*p be the pointer to the node after which we want to delete a new node.

```
void deleteAfterNode(NodeType *p)
{
    NodeType *q;
    if(p==NULL || p->next==NULL )
    {
        printf("Void insertion");
        exit(1);
    }
    else
    {
        q=p->next;
        p->next=q->next;
        free(q);
    }
}
```

### **An algorithm to delete a node at the specified position in a singly linked list:**

let \*head be the pointer to first node in the current list

1. Read position of a node which to be deleted, let it be pos.
2. if head==NULL
  - print "void deletion" and exit
3. Enter position of a node at which you want to delete a new node. Let this position is pos.
4. Set temp=head
  - declare a pointer of a structure let it be \*p
5. if (head ==NULL)then
  - print "void ideletion" and exit
  - otherwise;.
6. for(i=1; i<pos-1; i++)
  - temp=temp->next;
7. *print deleted item is temp->next->info*
8. *Set p=temp->next;*
9. Set temp->next =temp->next->next;
10. free(p);
11. End

### **The C function to delete a node at the specified position in a singly linked list**

```
void deleteAtSpecificPos()
{
    NodeType *temp *p;
    int pos, i;
    if(head==NULL)
    {
        printf("Empty list");
        return;
    }
    else
    {
        printf("Enter position of a node which you want to delete");
        scanf("%d", &pos);
```

```

temp=head;
for(i=1; i<pos-1; i++)
{
    temp=temp->next;
}
p=temp->next;
printf("Deleted item is %d", p->info);
temp->next =p->next;
free(p);
}
}

```

### **Searching an item in a linked list:**

To search an item from a given linked list we need to find the node that contain this data item.

If we find such a node then searching is successful otherwise searching unsuccessful.

*let \*head be the pointer to first node in the current list*

```

void searchItem()
{
    NodeType *temp;
    int key;
    if(head==NULL)
    {
        printf("empty list");
        exit(1);
    }
    else
    {
        printf("Enter searched item");
        scanf("%d",&key);
        temp=head;
        while(temp!=NULL)
        {
            if(temp->info==key)
            {
                printf("Search successful");
                break;
            }
            temp=temp->next;
        }
        if(temp==NULL)
            printf("Unsuccessful search");
    }
}

```

### **Complete program:**

***/\*\*\*\*\*\*Various operations on singly linked list\*\*\*\*\*\*/***

```

#include<stdio.h>
#include<conio.h>
#include<malloc.h> //for malloc function
#include<process.h> //for exit function
struct node
{

```

```

    int info;
    struct node *next;
};
typedef struct node NodeType;
NodeType *head;
head=NULL;
void insert_atfirst(int);
void insert_givenposition(int);
void insert_atend(int);
void delet_first();
void delet_last();
void delet_nthnode();
void info_sum();
void count_nodes();
void main()
{
    int choice;
    int item;
    clrscr();
    do
    {
        printf("\n manu for program:\n");
        printf("1. insert first \n2.insert at given position \n3 insert at last \n 4:Delete first
node\n 5:delete last node\n6:delete nth node\n7:count nodes\n8Display items\n10:exit\n");
        printf("enter your choice\n");
        scanf("%d",&choice);
        switch(choice)
        {
            case 1:
                printf("Enter item to be inserted");
                scanf("%d", &item)
                insert_atfirst(item);
                break;
            case 2:
                printf("Enter item to be inserted");
                scanf("%d", &item)
                insert_givenposition(item);
                break;
            case 3:
                printf("Enter item to be inserted");
                scanf("%d", &item)
                insert_atend();
                break;
            case 4:
                delet_first();
                break;
            case 5:
                delet_last();
                break;
            case 6:
                delet_nthnode();
                break;
            case 7:
                info_sum();
                break;
            case 8:

```

```

        count_nodes();
        break;
    case 9:
        exit(1);
        break;
    default:
        printf("invalid choice\n");
        break;
    }
} while(choice<10);
getch();
}

/*****function definitions*****/
void insert_atfirst(int item)
{
    NodeType *nnode;
    nnode=(NodeType*)malloc(sizeof(NodeType));
    nnode->info=item;
    nnode->next=head;
    head=nnode;
}

void insert_givenposition(int item)
{
    NodeType *nnode;
    NodeType *temp;
    temp=head;
    int p,i;
    nnode=( NodeType *)malloc(sizeof(NodeType));
    nnode->info=item;
    if (head==NULL)
    {
        nnode->next=NULL;
        head=nnode;
    }
    else
    {
        printf("Enter Position of a node at which you want to insert an new node\n");
        scanf("%d",&p);
        for(i=1;i<p-1;i++)
        {
            temp=temp->next;
        }
        nnode->next=temp->next;
        temp->next=nnode;
    }
}

void insert_atend(int item)
{
    NodeType *nnode;
    NodeType *temp;
    temp=head;
    nnode=( NodeType *)malloc(sizeof(NodeType));
    nnode->info=item;

```

```

    if(head==NULL)
    {
        nnode->next=NULL;
        head=nnode;
    }
    else
    {
        while(temp->next!=NULL)
        {
            temp=temp->next;
        }
        nnode->next=NULL;
        temp->next=nnode;
    }
}

void delet_first()
{
    NodeType *temp;
    if(head==NULL)
    {
        printf("Void deletion\n");
        return;
    }
    else
    {
        temp=head;
        head=head->next;
        free(temp);
    }
}

void delet_last()
{
    NodeType *hold,*temp;
    if(head==NULL)
    {
        printf("Void deletion\n");
        return;
    }
    else if(head->next==NULL)
    {
        hold=head;
        head=NULL;
        free(hold);
    }
    else
    {
        temp=head;
        while(temp->next->next!=NULL)
        {
            temp=temp->next;
        }
        hold=temp->next;
        temp->next=NULL;
        free(hold);
    }
}

```

```

}
void delet_nthnode()
{
    NodeType *hold,*temp;
    int pos, i;
    if(head==NULL)
    {
        printf("Void deletion\n");
        return;
    }
    else
    {
        temp=head;
        printf("Enter position of node which node is to be deleted\n");
        scanf("%d",&pos);
        for(i=1;i<pos-1;i++)
        {
            temp=temp->next;
        }
        hold=temp->next;
        temp->next=hold->next;
        free(hold);
    }
}
void info_sum()
{
    NodeType *temp;
    temp=head;
    while(temp!=NULL)
    {
        printf("%d\t",temp->info);
        temp=temp->next;
    }
}
void count_nodes()
{
    int cnt=0;
    NodeType *temp;
    temp=head;
    while(temp!=NULL)
    {
        cnt++;
        temp=temp->next;
    }
    printf("total nodes=%d",cnt);
}
}

```

### **Linked list implementation of Stack:**

#### **Push function:**

let \*top be the top of the stack or pointer to the first node of the list.

```

void push(item)
{
    NodeType *nnode;
    int data;

```

```

nnode=( NodeType *)malloc(sizeof(NodeType));
if(top==0)
{
    nnode->info=item;
    nnode->next=NULL;
    top=nnode;
}
else
{
    nnode->info=item;
    nnode->next=top;
    top=nnode;
}
}

```

**Pop function:**

let \*top be the top of the stack or pointer to the first node of the list.

```

void pop()
{
    NodeType *temp;
    if(top==0)
    {
        printf("Stack contain no elements:\n");
        return;
    }
    else
    {
        temp=top;
        top=top->next;
        printf("\ndeleted item is %d\t",temp->info);
        free(temp);
    }
}

```

**A Complete C program for linked list implementation of stack:**

/\*\*\*\*\*\*Linked list implementation of stack\*\*\*\*\*\*/

```

#include<stdio.h>
#include<conio.h>
#include<malloc.h>
#include<process.h>
struct node
{
    int info;
    struct node *next;
};
typedef struct node NodeType;
NodeType *top;
top=0;
void push(int);
void pop();
void display();
void main()
{
    int choice, item;
    clrscr();

```



```

do
{
    printf("\n1.Push \n2.Pop \n3.Display\n4.Exit\n");
    printf("enter ur choice\n");
    scanf("%d",&choice);
    switch(choice)
    {
        case 1:
            printf("\nEnter the data:\n");
            scanf("%d",&item);
            push(item);
            break;
        case 2:
            pop();
            break;
        case 3:
            display();
            break;
        case 4:
            exit(1);
            break;
        default:
            printf("invalid choice\n");
            break;
    }
} while(choice<5);
getch();
}

/*****push function*****/
void push(int item)
{
    NodeType *nnode;
    int data;
    nnode=( NodeType *)malloc(sizeof(NodeType));
    if(top==0)
    {
        nnode->info=item;
        nnode->next=NULL;
        top=nnode;
    }
    else
    {
        nnode->info=item;
        nnode->next=top;
        top=nnode;
    }
}

/*****pop function*****/
void pop()
{
    NodeType *temp;
    if(top==0)
    {
        printf("Stack contain no elements:\n");
        return;
    }
}

```

```

    }
    else
    {
        temp=top;
        top=top->next;
        printf("\ndeleted item is %d\t",temp->info);
        free(temp);
    }
}

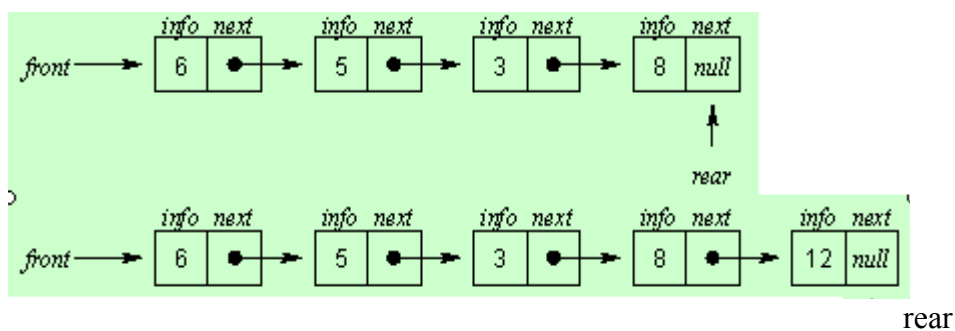
/*****display function*****/
void display()
{
    NodeType *temp;
    if(top==0)
    {
        printf("Stack is empty\n");
        return;
    }
    else
    {
        temp=top;
        printf("Stack items are:\n");
        while(temp!=0)
        {
            printf("%d\t",temp->info);
            temp=temp->next;
        }
    }
}

```

### **Linked list implementation of queue:**

#### **Insert function:**

let \*rear and \*front are pointers to the first node of the list initially and insertion of node in linked list done at the rear part and deletion of node from the linked list done from front part.



```

void insert(int item)
{
    NodeType *nnode;
    nnode=( NodeType *)malloc(sizeof(NodeType));

    if(rear==0)
    {
        nnode->info=item;
        nnode->next=NULL;
    }
}

```

```

        rear=front=nnode;
    }
    else
    {
        nnode->info=item;
        nnode->next=NULL;
        rear->next=nnode;
        rear=nnode;
    }
}

```

### **Delete function:**

let \*rear and \*front are pointers to the first node of the list initially and insertion of node in linked list done at the rear part and deletion of node from the linked list done from front part.

```

void delet()
{
    NodeType *temp;
    if(front==0)
    {
        printf("Queue contain no elements:\n");
        return;
    }
    else if(front->next==NULL)
    {
        temp=front;
        rear=front=NULL;
        printf("\nDeleted item is %d\n",temp->info);
        free(temp);
    }
    else
    {
        temp=front;
        front=front->next;
        printf("\nDeleted item is %d\n",temp->info);
        free(temp);
    }
}

```

### **A Complete C program for linked list implementation of queue:**

/\*\*\*\*\*\*Linked list implementation of queue\*\*\*\*\*\*/

```

#include<stdio.h>
#include<conio.h>
#include<malloc.h>
#include<process.h>
struct node
{
    int info;
    struct node *next;
};
typedef struct node NodeType;
NodeType *rear,*front;
rear=front=0;
void insert(int);

```

```

void delet();
void display();
void main()
{
    int choice, item;
    clrscr();
    do
    {
        printf("\n1.Insert \n2.Delet \n3.Display\n4.Exit\n");
        printf("enter ur choice\n");
        scanf("%d",&choice);
        switch(choice)
        {
            case 1:
                printf("\nEnter the data:\n");
                scanf("%d",&item);
                insert(item);
                break;
            case 2:
                delet();
                break;
            case 3:
                display();
                break;
            case 4:
                exit(1);
                break;
            default:
                printf("invalid choice\n");
                break;
        }
    } while(choice<5);
    getch();
}

/*****insert function*****/
void insert(int item)
{
    NodeType *nnode;
    nnode=( NodeType *)malloc(sizeof(NodeType));

    if(rear==0)
    {
        nnode->info=item;
        nnode->next=NULL;
        rear=front=nnode;
    }
    else
    {
        nnode->info=item;
        nnode->next=NULL;
        rear->next=nnode;
        rear=nnode;
    }
}

/*****delet function*****/

```

```

void delet()
{
    NodeType *temp;
    if(front==0)
    {
        printf("Queue contain no elements:\n");
        return;
    }
    else if(front->next==NULL)
    {
        temp=front;
        rear=front=NULL;
        printf("\nDeleted item is %d\n",temp->info);
        free(temp);
    }
    else
    {
        temp=front;
        front=front->next;
        printf("\nDeleted item is %d\n",temp->info);
        free(temp);
    }
}

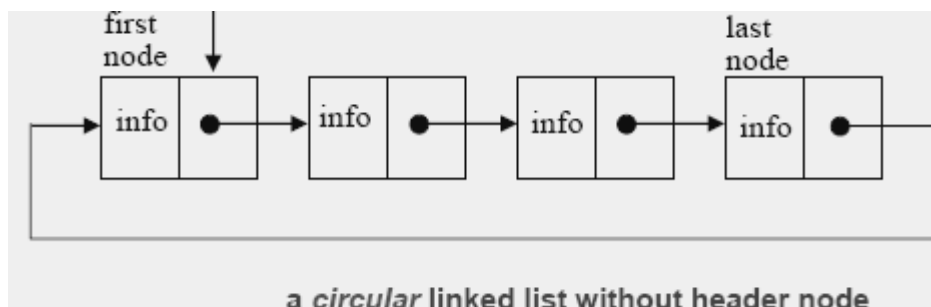
/*****display function*****/
void display()
{
    NodeType *temp;
    temp=front;
    printf("\nqueue items are:\t");
    while(temp!=NULL)
    {
        printf("%d\t",temp->info);
        temp=temp->next;
    }
}

```

### **Circular Linked list:**

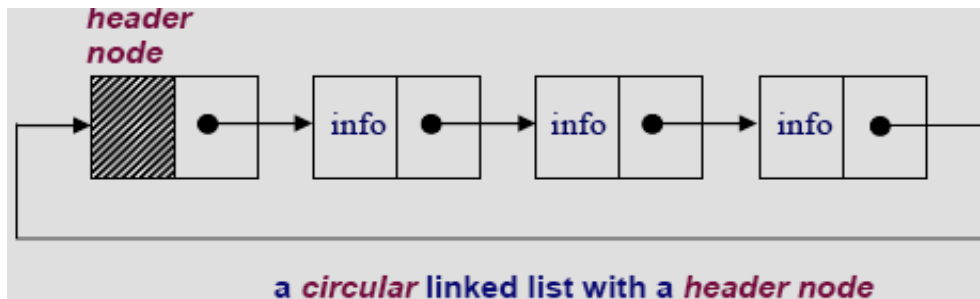
A circular linked list is a list where the link field of last node points to the very first node of the list .

Circular linked lists can be used to help the traverse the same list again and again if needed. A circular list is very similar to the linear list where in the circular list the pointer of the last node points not NULL but the first node.

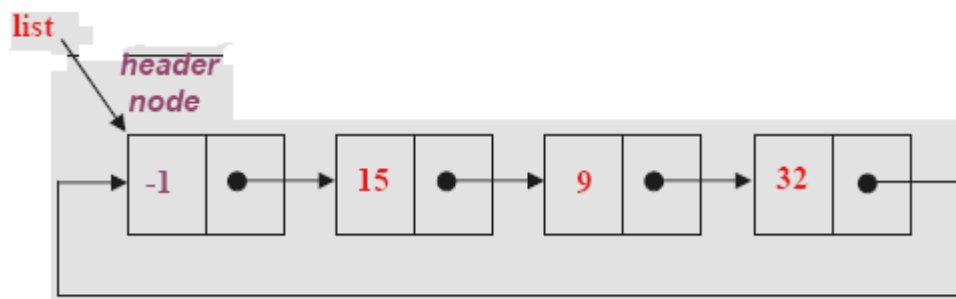


In a circular linked list there are two methods to know if a node is the first node or not.

- ◆ Either an external pointer, **list**, points the first node or
  - ◆ A **header node** is placed as the first node of the circular list.
- The header node can be separated from the others by either having a **sentinel value** as the info part or having a dedicated **flag** variable to specify if the node is a header node or not.



### CIRCULAR LIST with header node



### C representation of circular linked list:

we declare the structure for the circular linked list in the same way as declared it for the linear linked list.

```
struct node
{
    int info;
    struct node *next;
};
typedef struct node NodeType;
NodeType *start=NULL;
NodeType *last=NULL;
```

### Algorithms to insert a node in a circular linked list:

#### Algorithm to insert a node at the beginning of a circular linked list:

1. Create a new node as  
`newnode=(NodeType*)malloc(sizeof(NodeType));`
2. if `start==NULL` then  
     set `newnode->info=item`  
     set `newnode->next=newnode`  
     set `start=newnode`  
     set `last newnode`  
 end if
3. else  
     set `newnode->info=item`  
     set `newnode->next=start`  
     set `start=newnode`  
     set `last->next=newnode`

```

    end else
4. End

```

**Algorithm to insert a node at the end of a circular linked list:**

```

1. Create a new node as
   newnode=(NodeType*)malloc(sizeof(NodeType));
2. if start==NULL then
   set newnode->info=item
   set newnode->next=newnode
   set start=newnode
   set last newnode
   end if
3. else
   set newnode->info=item
   set last->next=newnode
   set last=newnode
   set last->next=start
   end else
4. End

```

**C function to insert a node at the beginning of a circular linked list:**

```

void InsertAtBeg(int Item)
{
    NodeType *newnode;
    newnode=(NodeType*)malloc(sizeof(NodeType));
    if(start==NULL)
    {
        newnode->info=item;
        newnode->next=newnode;
        start=newnode;
        last newnode;
    }
    else
    {
        newnode->info=item;
        last->next=newnode;
        last=newnode;
        last->next=start;
    }
}

```

**C function to insert a node at the end of a circular linked list:**

```

void InsertAtEnd(int Item)
{
    NodeType *newnode;
    newnode=(NodeType*)malloc(sizeof(NodeType));
    if(start==NULL)
    {
        newnode->info=item;
        newnode->next=newnode;
        start=newnode;
        last newnode;
    }
    else
    {

```

```

newnode->info=item;
last->next=newnode;
last=newnode;
last->next=start;

    }
}

```

### **Algorithms to delete a node from a circular linked list:**

#### **Algorithm to delete a node from the beginning of a circular linked list:**

1. if start==NULL then  
    “empty list” and exit
2. else  
    set temp=start  
    set start=start->next  
    print the deleted element=temp->info  
    set last->next=start;  
    free(temp)  
  end else
3. End

#### **Algorithm to delete a node from the end of a circular linked list:**

1. if start==NULL then  
    “empty list” and exit
2. else if start==last  
    set temp=start  
    print deleted element=temp->info  
    free(temp)  
    start=last=NULL
3. else  
    set temp=start  
    while( temp->next!=last)  
        set temp=temp->next  
    end while  
    set hold=temp->next  
    set last=temp  
    set last->next=start  
    print the deleted element=hold->info  
    free(hold)  
  end else
4. End

#### **C function to delete a node from the beginning of a circular linked list:**

```

void DeleteFirst()
{
    if(start==NULL)
    {
        printf("Empty list");
        exit(1);
    }
    else
    {
        temp=start;
        start=start->next;
        printf(" the deleted element=%d", temp->info);
        last->next=start;
        free(temp)
    }
}

```



```

    }
}

```

**C function to delete a node from the end of a circular linked list:**

```

void DeleteLast()
{
    if(start==NULL)
    {
        printf("Empty list");
        exit(1);
    }
    else if(start==last) //for only one node
    {
        temp=start;
        printf("deleted element=%d", temp->info);
        free(temp);
        start=last=NULL;
    }
    else
    {
        temp=start;
        while( temp->next!=last)
            temp=temp->next;
        hold=temp->next;
        last=temp;
        last->next=start;
        printf("the deleted element=%d", hold->info);
        free(hold);
    }
}

```

**Stack as a circular List:**

To implement a stack in a circular linked list, let pstack be a pointer to the last node of a circular list. Actually there is no any end of a list but for convention let us assume that the first node(rightmost node of a list) is the top of the stack.

An empty stack is represented by a null list.

The structure for the circular linked list implementation of stack is:

```

struct node
{
    int info;
    struct node *next;
};
typedef struct node NodeType;
NodeType *pstack=NULL;

```

**C function to check whether the list is empty or not as follows:**

```

int IsEmpty()
{
    if(pstack==NULL)
        return(1);
    else
        return(0);
}

```

**PUSH function:**

```

void PUSH(int item)
{
    NodeType newnode;
    newnode=(NodeType*)malloc(sizeof(NodeType));
    newnode->info=item;
    if(pstack==NULL)
    {
        pstack=newnode;
        pstack->next=pstack;
    }
    else
    {
        newnode->next=pstack->next;
        pstack->next=newnode;
    }
}

```

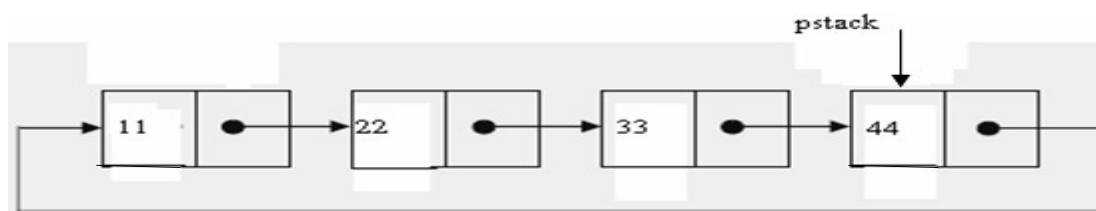
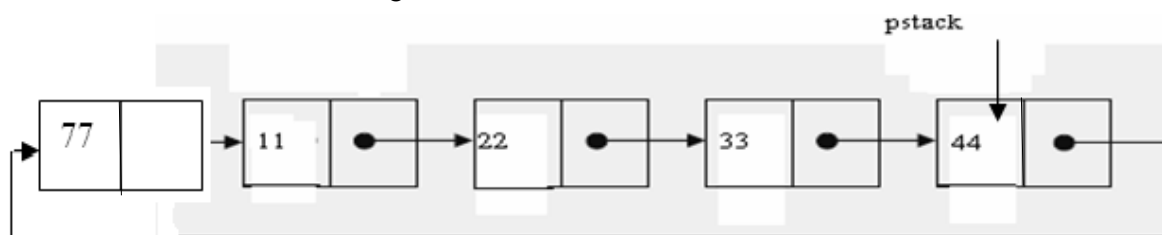


fig: circular linked list

**POP function:**

```

void POP()
{
    NodeType *temp;
    if(pstack==NULL)
    {
        printf("Stack underflow\n");
        exit(1);
    }
    else if(pstack->next==pstack) //for only one node
    {
        printf("popped item=%d", pstack->info);
        pstack=NULL;
    }
    else
    {
        temp=pstack->next;
        pstack->next=temp->next;
        printf("popped item=%d", temp->info);
        free(temp);
    }
}

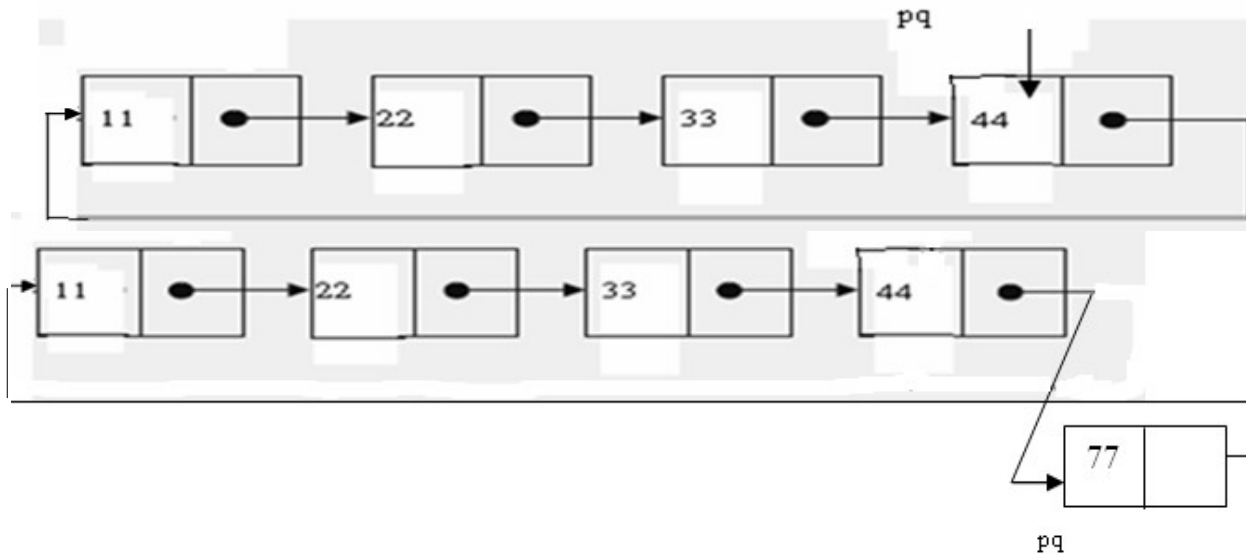
```

**Queue as a circular List:**

It is easier to represent a queue as a circular list than as a linear list. As a linear list a queue is specified by two pointers, one to the front of the list and the other to its rear. However, by using a circular list, a queue may be specified by a single pointer q to that list. node(q) is the rear of the queue and the following node is its front.

**Insertion function:**

```
void insert(int item)
{
    NodeType *nnode;
    nnode=( NodeType *)malloc(sizeof(NodeType));
    nnode->info=item;
    if(pq==NULL)
        pq=nnode;
    else
    {
        nnode->next=pq->next;
        pq->next=nnode;
        pq=nnode;
    }
}
```

**Deletion function:**

```
void delet(int item)
{
    NodeType *temp;
    if(pq==NULL)
    {
        printf("void deletion\n");
        exit(1);
    }
    else if(pq->next==pq) //for only one node
    {
        printf("poped item=%d", pq->info);
        pq=NULL;
    }
}
```

```

else
{
    temp=pq->next;
    pq->next=temp->next;
    printf("popped item=%d", temp->info);
    free(temp);
}
}

```

### **Doubly Linked List:**

A linked list in which all nodes are linked together by multiple number of links ie each node contains three fields (two pointer fields and one data field) rather than two fields is called doubly linked list.

It provides bidirectional traversal.

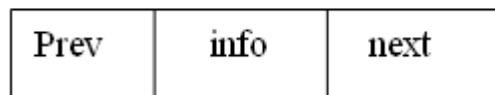


Fig: A node in doubly linked list

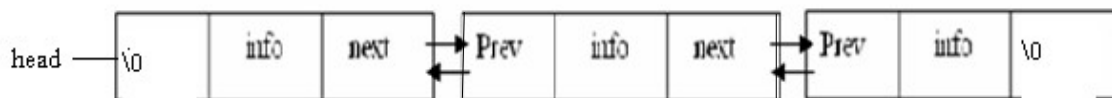


fig: A doubly linked list with three nodes

### **C representation of doubly linked list:**

```

struct node
{
    int info;
    struct node *prev;
    struct node *next;
};
typedef struct node NodeType;
NodeType *head=NULL;

```

### **Algorithms to insert a node in a doubly linked list:**

#### **Algorithm to insert a node at the beginning of a doubly linked list:**

1. Allocate memory for the new node as,  
newnode=(NodeType\*)malloc(sizeof(NodeType))
2. Assign value to info field of a new node  
set newnode->info=item
3. set newnode->prev=newnode->next=NULL
4. set newnode->next=head
5. set head->prev=newnode
6. set head=newnode
7. End

### **C function to insert a node at the beginning of a doubly linked list:**

```

void InsertAtBeg(int Item)
{
    NodeType *newnode;
    newnode=(NodeType*)malloc(sizeof(NodeType));
    newnode->info=item;
    newnode->prev=newnode->next=NULL;
}

```

```

newnode->next=head;
head->prev=newnode;
head=newnode;
}

```

**Algorithm to insert a node at the end of a doubly linked list:**

1. Allocate memory for the new node as,  
newnode=(NodeType\*)malloc(sizeof(NodeType))
2. Assign value to info field of a new node  
set newnode->info=item
3. set newnode->next=NULL
4. if head==NULL  
set newnode->prev=NULL;  
set head=newnode;
5. if head!=NULL  
set temp=head  
while(temp->next!=NULL)  
temp=temp->next;  
end while  
set temp->next=newnode;  
set newnode->prev=temp
6. End

**Algorithm to delete a node from beginning of a doubly linked list:**

1. if head==NULL then  
print “empty list” and exit
2. else  
set hold=head  
set head=head->next  
set head->prev=NULL;  
free(hold)
3. End

**Algorithm to delete a node from end of a doubly linked list:**

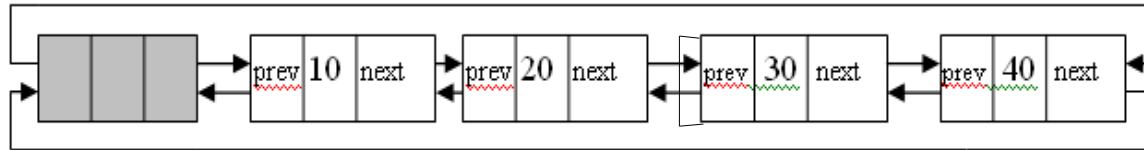
1. if head==NULL then  
print “empty list” and exit
2. else if(head->next==NULL) then  
set hold=head  
set head=NULL  
free(hold)
3. else  
set temp=head;  
while(temp->next->next !=NULL)  
temp=temp->next  
end while  
set hold=temp->next  
set temp->next=NULL  
free(hold)
4. End

**Circular Doubly Linked List:**

A circular doubly linked list is one which has the successor and predecessor pointer in circular manner.

It is a doubly linked list where the next link of last node points to the first node and previous link of first node points to last node of the list.

The main objective of considering circular doubly linked list is to simplify the insertion and deletion operations performed on doubly linked list.



head node

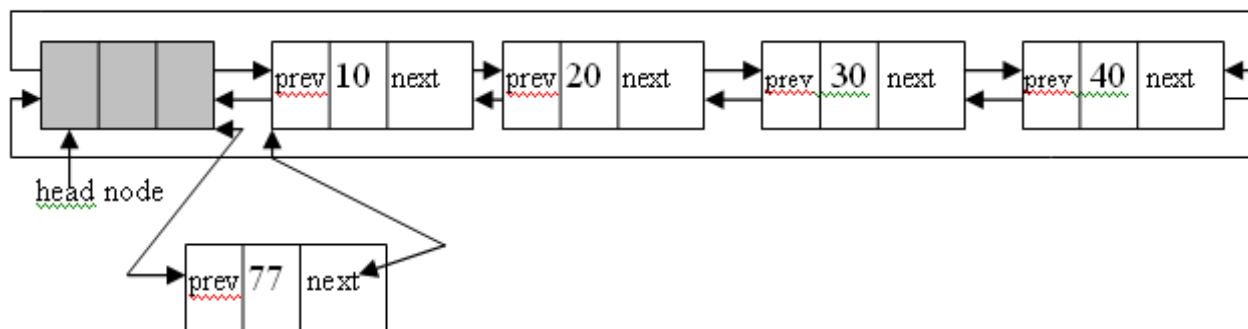
Fig: A circular doubly linked list

### **C representation of doubly circular linked list:**

```
struct node
{
    int info;
    struct node *prev;
    struct node *next;
};
typedef struct node NodeType;
NodeType *head=NULL;
```

#### **Algorithm to insert a node at the beginning of a circular doubly linked list:**

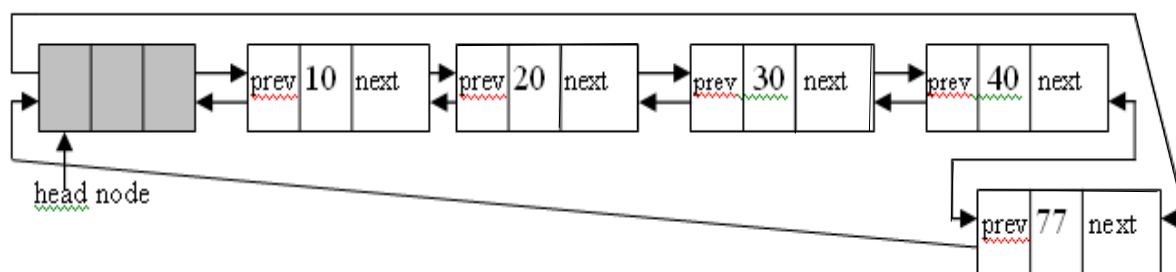
1. Allocate memory for the new node as,  
newnode=(NodeType\*)malloc(sizeof(NodeType))
2. Assign value to info field of a new node  
set newnode->info=item
3. set temp=head->next
4. set head->next=newnode
5. set newnode->prev=head
6. set newnode->next=temp
7. set temp->prev=newnode
8. End



#### **Algorithm to insert a node at the end of a circular doubly linked list:**

1. Allocate memory for the new node as,  
newnode=(NodeType\*)malloc(sizeof(NodeType))
2. Assign value to info field of a new node  
set newnode->info=item
3. set temp=head->prev
4. set temp->next=newnode
5. set newnode->prev=temp
6. set newnode->next=head

7. set head->prev=newnode
8. End



**Algorithm to delete a node from the beginning of a circular doubly linked list:**

1. if head->next==NULL then  
    print "empty list" and exit
2. else  
    set temp=head->next;  
    set head->next=temp->next  
    set temp->next=head  
    free(temp)
3. End

**Algorithm to delete a node from the end of a circular doubly linked list:**

1. if head->next==NULL then  
    print "empty list" and exit
2. else  
    set temp=head->prev;  
    set head->left=temp->left  
    free(temp)
3. End

## **Unit 6**

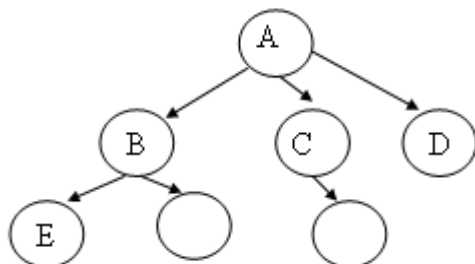
### **Tree data structure:**

- a) Concept and definition
- b) Binary tree
- c) Introduction and application
- d) operations
- e) Types of binary tree
  - ✓ Complete binary tree
  - ✓ Strictly binary tree
  - ✓ Almost complete binary tree
- f) Huffman algorithm
- g) Binary search tree
  - ✓ insertion
  - ✓ deletion
  - ✓ searching
- h) Tree traversal
  - ✓ Pre-order traversal
  - ✓ In-order traversal
  - ✓ post-order traversal

**Tree:**

A tree is an abstract model of a hierarchical structure that consists of nodes with a parent-child relationship.

- Tree is a sequence of nodes.
- There is a starting node known as root node.
- Every node other than the root has a parent node.
- Nodes may have any number of children.



A has 3 children, B, C, D  
A is parent of B

**Recursive definition of tree:**

A tree  $t$  of order  $n$  is either empty or consists of a distinguished node  $r$ , called the root of  $T$ , together with at most  $n$  trees,  $T_1, T_2, \dots, T_n$  called the sub trees of  $T$ .

**Characteristics of trees:**

- ✓ Non-linear data structure
- ✓ combines advantages of an ordered array
- ✓ searching as fast as in ordered array
- ✓ insertion and deletion as fast as in linked list

**Application:**

- ✓ Directory structure of a file store
- ✓ Structure of arithmetic expressions
- ✓ Hierarchy of an organization

**Some key terms:****Degree of a node:**

The degree of a node is the number of children of that node.  
In above tree the degree of node A is 3.

**Degree of a Tree:**

The degree of a tree is the maximum degree of nodes in a given tree.

In the above tree the node A has maximum degree, thus the degree of the tree is 3.



**Path:**

It is the sequence of consecutive edges from source node to destination node.  
There is a single unique path from the root to any node.

**Height of a node:**

The height of a node is the maximum path length from that node to a leaf node. A leaf node has a height of 0.

**Height of a tree:**

The height of a tree is the height of the root.

**Depth of a node:**

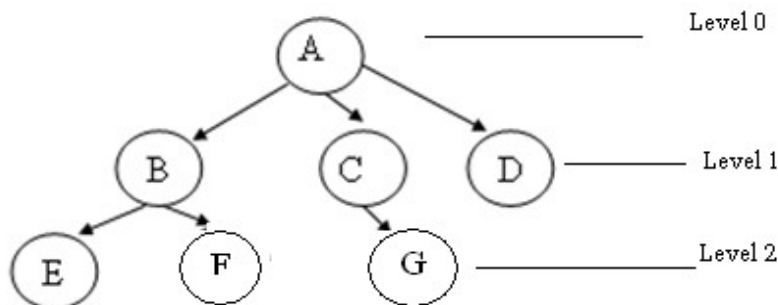
Depth of a node is the path length from the root to that node. The root node has a depth of 0.

**Depth of a tree:**

Depth of a tree is the maximum level of any leaf in the tree.  
This is equal to the longest path from the root to any leaf.

**Level of a node:**

the level of a node is 0, if it is root; otherwise it is one more than its parent.

**Illustration:**

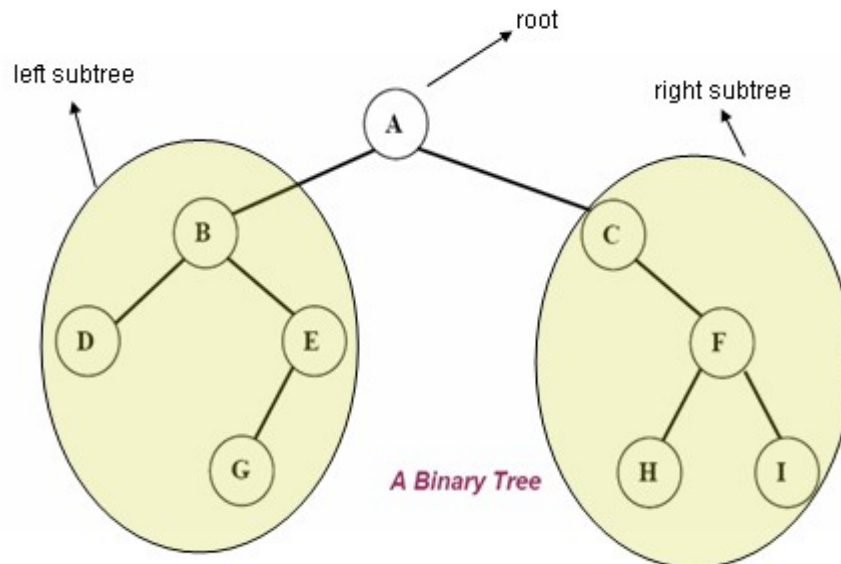
- ✓ A is the root node
- ✓ B is the parent of E and F
- ✓ D is the sibling of B and C
- ✓ E and F are children of B
- ✓ E, F, G, D are external nodes or leaves
- ✓ A, B, C are internal nodes
- ✓ Depth of F is 2
- ✓ the height of tree is 2
- ✓ the degree of node A is 3
- ✓ The degree of tree is 3

**Binary Trees:**

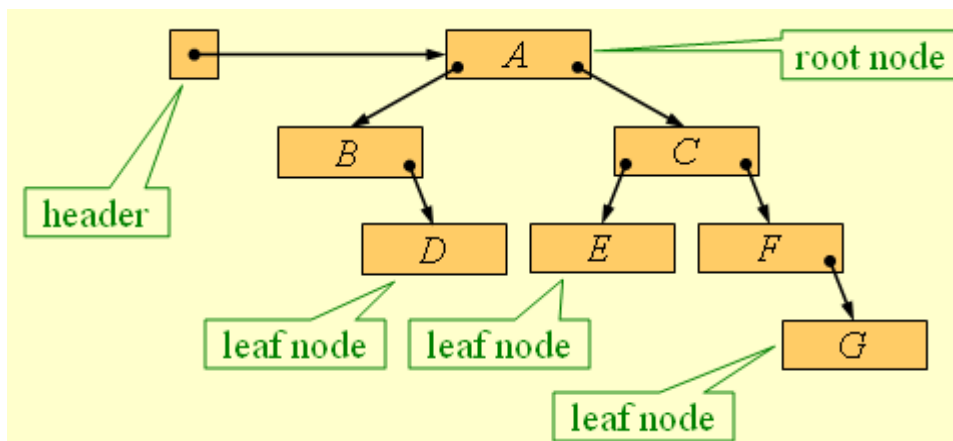
A binary tree is a finite set of elements that are either empty or is partitioned into three disjoint subsets. The first subset contains a single element called the **root** of the tree. The other

two subsets are themselves binary trees called the **left** and **right sub-trees** of the original tree. A left or right sub tree can be empty.

Each element of a binary tree is called a **node** of the tree. The following figure shows a binary tree with 9 nodes where A is the root.



• A **binary tree** consists of a **header**, plus a number of **nodes** connected by **links** in a hierarchical data structure:



### Binary tree properties:

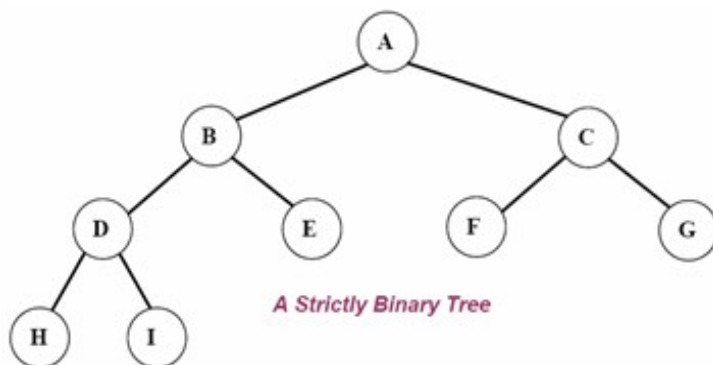
- ✓ If a binary tree contains  $m$  nodes at level  $l$ , it contains at most  $2m$  nodes at level  $l+1$ .
- ✓ Since a binary tree can contain at most 1 node at level 0 (the root), it contains at most  $2^l$  nodes at level  $l$ .

### Types of binary tree

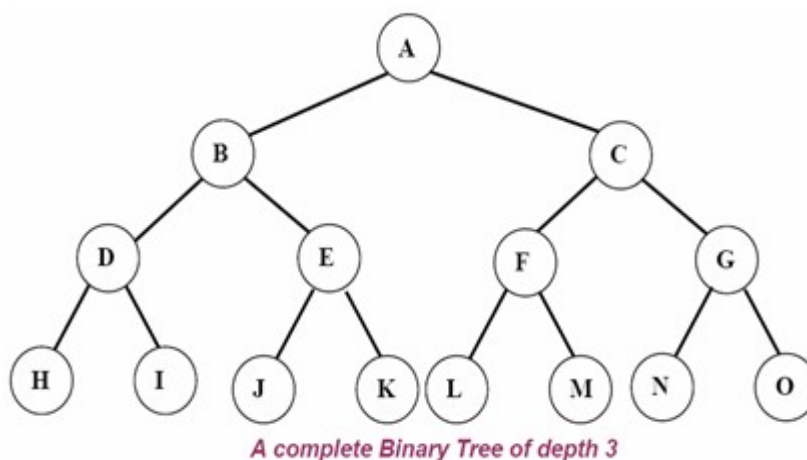
- ✓ Complete binary tree
- ✓ Strictly binary tree
- ✓ Almost complete binary tree

**Strictly binary tree:**

If every non-leaf node in a binary tree has nonempty left and right sub-trees, then such a tree is called a *strictly binary tree*.

**Complete binary tree:**

A *complete binary tree* of depth  $d$  is called strictly binary tree if all of whose leaves are at level  $d$ . A complete binary tree with depth  $d$  has  $2^d$  leaves and  $2^d - 1$  non-leaf nodes(internal)

**Almost complete binary tree:**

A binary tree of depth  $d$  is an almost complete binary tree if:

- ✓ Any node  $nd$  at level less than  $d-1$  has two sons.
- ✓ For any node  $nd$  in the tree with a right descendant at level  $d$ ,  $nd$  must have a left son and every left descendant of  $nd$  is either a leaf at level  $d$  or has two sons.

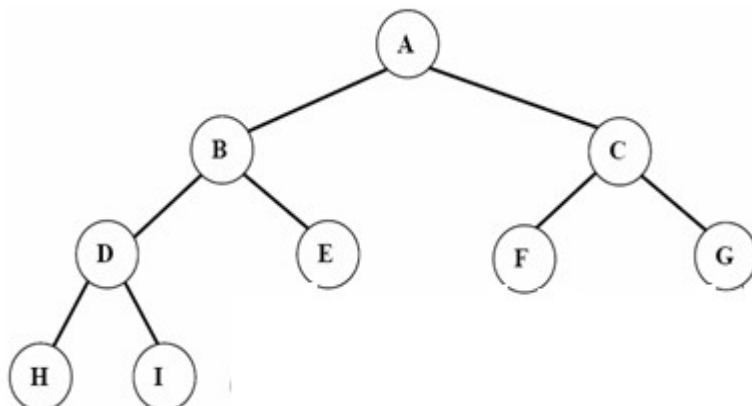


Fig Almost complete binary tree.

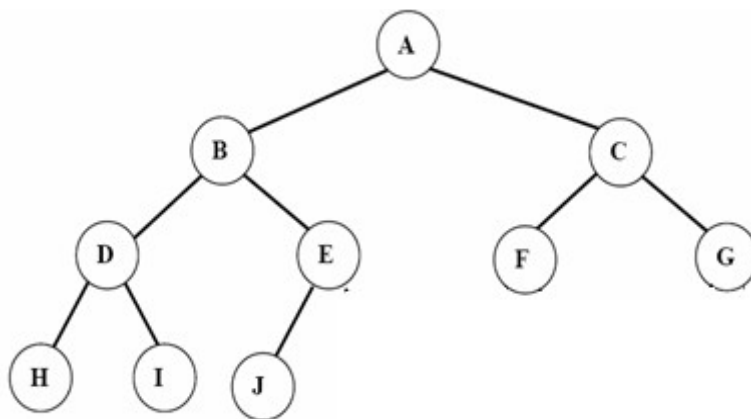


Fig Almost complete binary tree but not strictly binary tree.  
Since node E has a left son but not a right son.

### Operations on Binary tree:

- ✓ **father(n,T):** Return the parent node of the node n in tree T. If n is the root, NULL is returned.
- ✓ **LeftChild(n,T):** Return the left child of node n in tree T. Return NULL if n does not have a left child.
- ✓ **RightChild(n,T):** Return the right child of node n in tree T. Return NULL if n does not have a right child.
- ✓ **Info(n,T):** Return information stored in node n of tree T (ie. Content of a node).
- ✓ **Sibling(n,T):** return the sibling node of node n in tree T. Return NULL if n has no sibling.
- ✓ **Root(T):** Return root node of a tree if and only if the tree is nonempty.
- ✓ **Size(T):** Return the number of nodes in tree T
- ✓ **MakeEmpty(T):** Create an empty tree T
- ✓ **SetLeft(S,T):** Attach the tree S as the left sub-tree of tree T
- ✓ **SetRight(S,T):** Attach the tree S as the right sub-tree of tree T.
- ✓ **Preorder(T):** Traverses all the nodes of tree T in preorder.
- ✓ **postorder(T):** Traverses all the nodes of tree T in postorder
- ✓ **Inorder(T):** Traverses all the nodes of tree T in inorder.

### C representation for Binary tree:

```

struct bnode
{
    int info;
    struct bnode *left;
    struct bnode *right;
};
struct bnode *root=NULL;
  
```

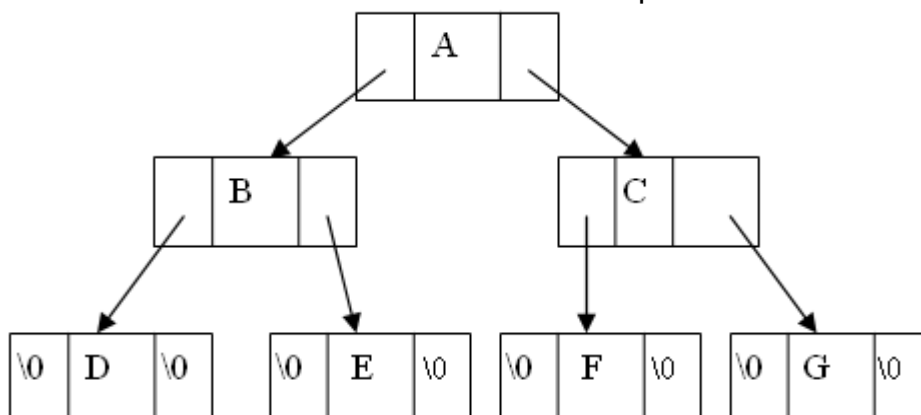


Fig: Structure of Binary tree

**Tree traversal:**

The tree traversal is a way in which each node in the tree is visited exactly once in a symmetric manner.

There are three popular methods of traversal

- ✓ Pre-order traversal
- ✓ In-order traversal
- ✓ Post-order traversal

**Pre-order traversal:**

The preorder traversal of a nonempty binary tree is defined as follows:

- ✓ Visit the root node
- ✓ Traverse the left sub-tree in preorder
- ✓ Traverse the right sub-tree in preorder

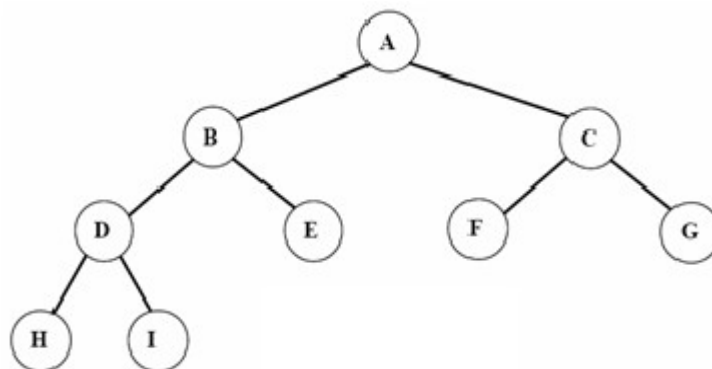


fig Binary tree

The preorder traversal output of the given tree is: A B D H I E C F G

The preorder is also known as depth first order.

**C function for preorder traversing:**

```

void preorder(struct bnode *root)
{
    if(root!=NULL)
    {
        printf("%c", root->info);
        preorder(root->left);
        preorder(root->right);
    }
}
  
```

**In-order traversal:**

The inorder traversal of a nonempty binary tree is defined as follows:

- ✓ Traverse the left sub-tree in inorder
- ✓ Visit the root node
- ✓ Traverse the right sub-tree in inorder

The inorder traversal output of the given tree is: H D I B E A F C G

**C function for inorder traversing:**

```
void inorder(struct bnode *root)
{
    if(root!=NULL)
    {
        inorder(root->left);
        printf("%c", root->info);
        inorder(root->right);
    }
}
```

**Post-order traversal:**

The post-order traversal of a nonempty binary tree is defined as follows:

- ✓ Traverse the left sub-tree in post-order
- ✓ Traverse the right sub-tree in post-order
- ✓ Visit the root node

The post-order traversal output of the given tree is: H I D E B F G C A

**C function for post-order traversing:**

```
void post-order(struct bnode *root)
{
    if(root!=NULL)
    {
        post-order(root->left);
        post-order(root->right);
        printf("%c", root->info);
    }
}
```

**Binary search tree(BST):**

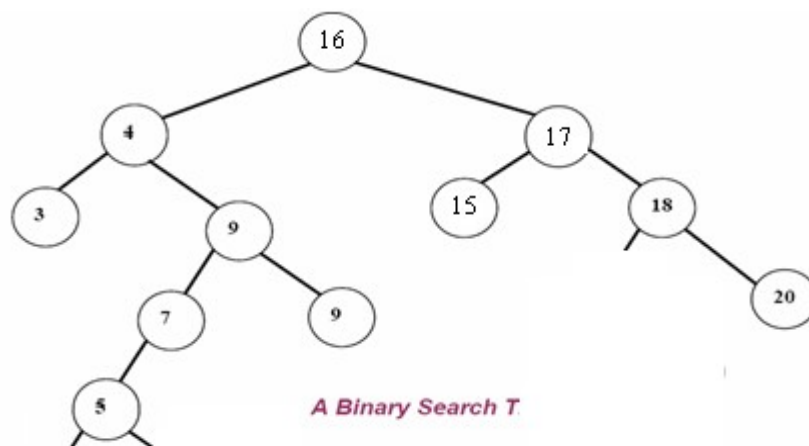
A binary search tree (BST) is a binary tree that is either empty or in which every node contains a key (value) and satisfies the following conditions:

- ✓ All keys in the left sub-tree of the root are smaller than the key in the root node
- ✓ All keys in the right sub-tree of the root are greater than the key in the root node
- ✓ The left and right sub-trees of the root are again binary search trees

Given the following sequence of numbers,

14, 15, 4, 9, 7, 18, 3, 5, 16, 4, 20, 17, 9, 14, 5

The following binary search tree can be constructed:



## Operations on Binary search tree(BST):

Following operations can be done in BST:

- ✓ **Search(k, T):** Search for key k in the tree T. If k is found in some node of tree then return true otherwise return false.
- ✓ **Insert(k, T):** Insert a new node with value k in the info field in the tree T such that the property of BST is maintained.
- ✓ **Delete(k, T):** Delete a node with value k in the info field from the tree T such that the property of BST is maintained.
- ✓ **FindMin(T), FindMax(T):** Find minimum and maximum element from the given nonempty BST.

## Searching through the BST:

- **Problem:** Search for a given target value in a BST.
- **Idea:** Compare the target value with the element in the root node.
  - ✓ If the target value is **equal**, the search is successful.
  - ✓ If target value is **less**, search the left subtree.
  - ✓ If target value is **greater**, search the right subtree.
  - ✓ If the subtree is **empty**, the search is unsuccessful.

## BST search algorithm:

To find which if any node of a BST contains an element equal to *target*:

1. Set *curr* to the BST's root.
2. Repeat:
  - 2.1. If *curr* is null:
    - 2.1.1. Terminate with answer *none*.
  - 2.2. Otherwise, if *target* is equal to *curr*'s element:
    - 2.2.1. Terminate with answer *curr*.
  - 2.3. Otherwise, if *target* is less than *curr*'s element:
    - 2.3.1. Set *curr* to *curr*'s left child.
  - 2.4. Otherwise, if *target* is greater than *curr*'s element:
    - 2.4.1. Set *curr* to *curr*'s right child.

2. end

**C function for BST searching:**

```

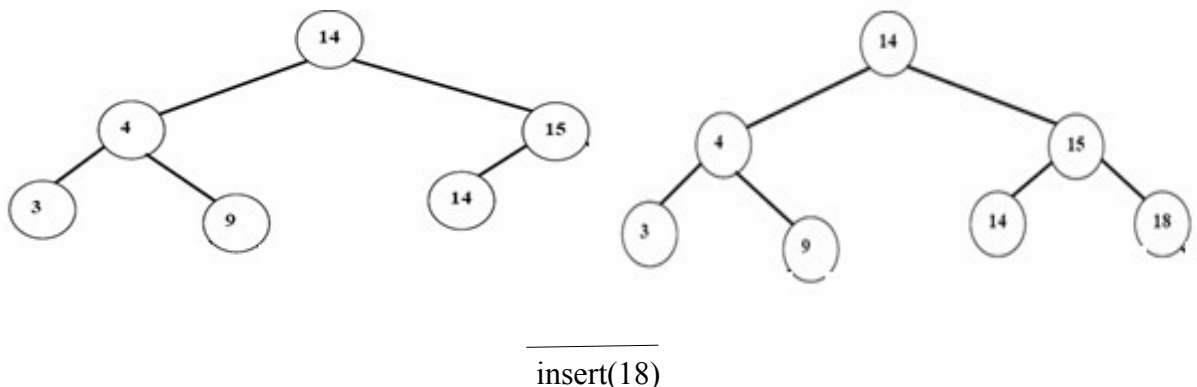
void BinSearch(struct bnode *root , int key)
{
    if(root == NULL)
    {
        printf("The number does not exist");
        exit(1);
    }
    else if (key == root->info)
    {
        printf("The searched item is found");
    }
    else if(key < root->info)
        return BinSearch(root->left, key);
    else
        return BinSearch(root->right, key);
}

```

**Insertion of a node in BST:**

To insert a new item in a tree, we must first verify that its key is different from those of existing elements. To do this a search is carried out. If the search is unsuccessful, then item is inserted.

•**Idea:** To insert a new element into a BST, proceed as if searching for that element. If the element is not already present, the search will lead to a null link. Replace that null link by a link to a leaf node containing the new element.

**BST insertion algorithm:**

To insert the element *elem* into a BST:

1. Set *parent* to null, and set *curr* to the BST's root.
2. Repeat:
  - 2.1. If *curr* is null:
    - 2.1.1. Replace the null link from which *curr* was taken (either the BST's root or *parent*'s left child or *parent*'s right child) by a link to a newly-created leaf node with element *elem*.
    - 2.1.2. Terminate.
  - 2.2. Otherwise, if *elem* is equal to *curr*'s element:
    - 2.2.1. Terminate.
  - 2.3. Otherwise, if *elem* is less than *curr*'s element:
    - 2.3.1. Set *parent* to *curr*, and set *curr* to *curr*'s left child.
  - 2.4. Otherwise, if *elem* is greater than *curr*'s element:
    - 2.4.1. Set *parent* to *curr*, and set *curr* to *curr*'s right child.
3. End



**C function for BST insertion:**

```

void insert(struct bnode *root, int item)
{
    if(root=NULL)
    {
        root=(struct bnode*)malloc (sizeof(struct bnode));
        root->left=root->right=NULL;
        root->info=item;
    }
    else
    {
        if(item<root->info)
            root->left=insert(root->left, item);
        else
            root->right=insert(root->right, item);
    }
}

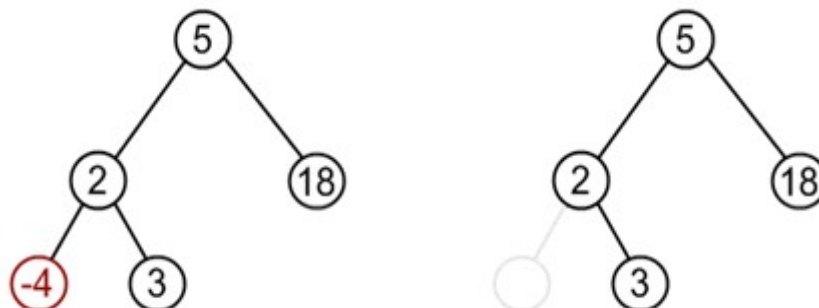
```

**Deleting a node from the BST:**

While deleting a node from BST, there may be three cases:

***1. The node to be deleted may be a leaf node:***

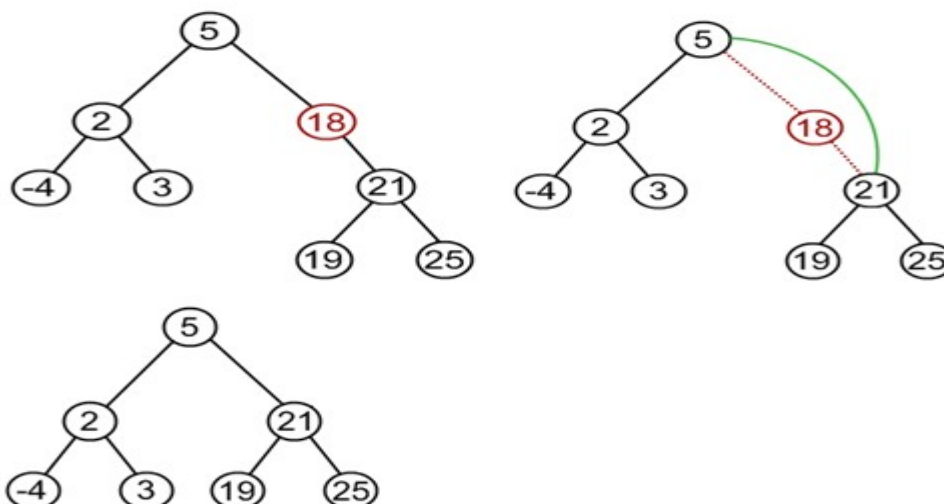
In this case simply delete a node and set null pointer to its parents those side at which this deleted node exist.



Suppose node to be deleted is -4

***2. The node to be deleted has one child:***

In this case the child of the node to be deleted is appended to its parent node.  
Suppose node to be deleted is 18

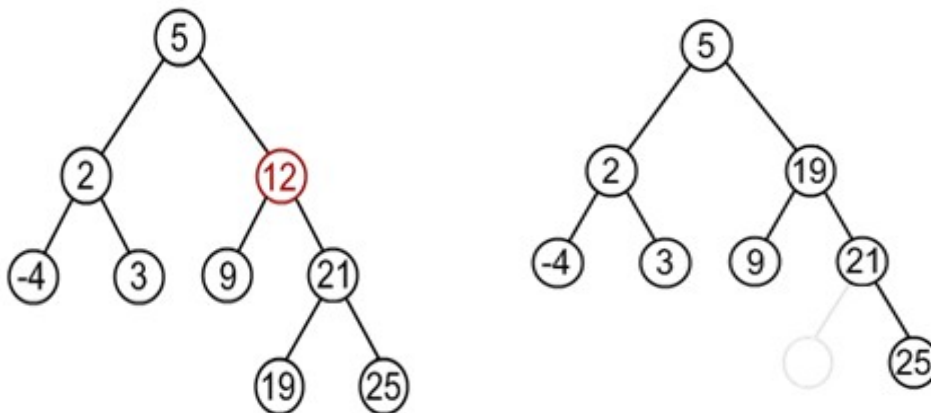


**3. the node to be deleted has two children:**

In this case node to be deleted is replaced by its in-order successor node.

OR

If the node to be deleted is either replaced by its right sub-trees leftmost node or its left sub-trees rightmost node.



Suppose node to be deleted is 12

Find minimum element in the right sub-tree of the node to be removed. In current example it is 19.

**General algorithm to delete a node from a BST:**

1. start
2. if a node to be deleted is a leaf node at left side then simply delete and set null pointer to it's parent's left pointer.
3. If a node to be deleted is a leaf node at right side then simply delete and set null pointer to it's parent's right pointer
4. if a node to be deleted has one child then connect it's child pointer with it's parent pointer and delete it from the tree
5. if a node to be deleted has two children then replace the node being deleted either by
  - a. right most node of it's left sub-tree or
  - b. left most node of it's right sub-tree.
6. End

**The deleteBST function:**

```

struct bnode *delete(struct bnode *root, int item)
{
    struct bnode *temp;
    if(root==NULL)
    {
        printf("Empty tree");
        return;
    }
    else if(item<root->info)
        root->left=delete(root->left, item);
    else if(item>root->info)
        root->right=delete(root->right, item);
    else if((root->left!=NULL && root->right!=NULL) //node has two child
    {
        temp=find_min(root->right);
        root->info=temp->info;
        root->right=delete(root->right, root->info);
    }
}
  
```

```

    }
    else
    {
        temp=root;
        if(root->left==NULL)
            root=root->right;
        else if(root->right==NULL)
            root=root->left;
        free(temp);
    }
    return(temp);
}
/*****find minimum element function*****/
struct bnode *find_min(struct bnode *root)
{
    if(root==NULL)
        return 0;
    else if(root->left==NULL)
        return root;
    else
        return(find_min(root->left));
}

```

## **Huffman algorithm:**

*Huffman algorithm is a method for building an extended binary tree with a minimum weighted path length from a set of given weights.*

- This is a method for the construction of minimum redundancy codes .
- Applicable to many forms of data transmission



*Our example: text files*

-1951, David Huffman found the “most efficient method of representing numbers, letters, and other symbols using binary code”. Now standard method used for data compression.

In Huffman Algorithm, a set of nodes assigned with values if fed to the algorithm. Initially 2 nodes are considered and their sum forms their parent node. When a new element is considered, it can be added to the tree. Its value and the previously calculated sum of the tree are used to form the new node which in turn becomes their parent.

Let us take any four characters and their frequencies, and *sort* this list by increasing frequency.

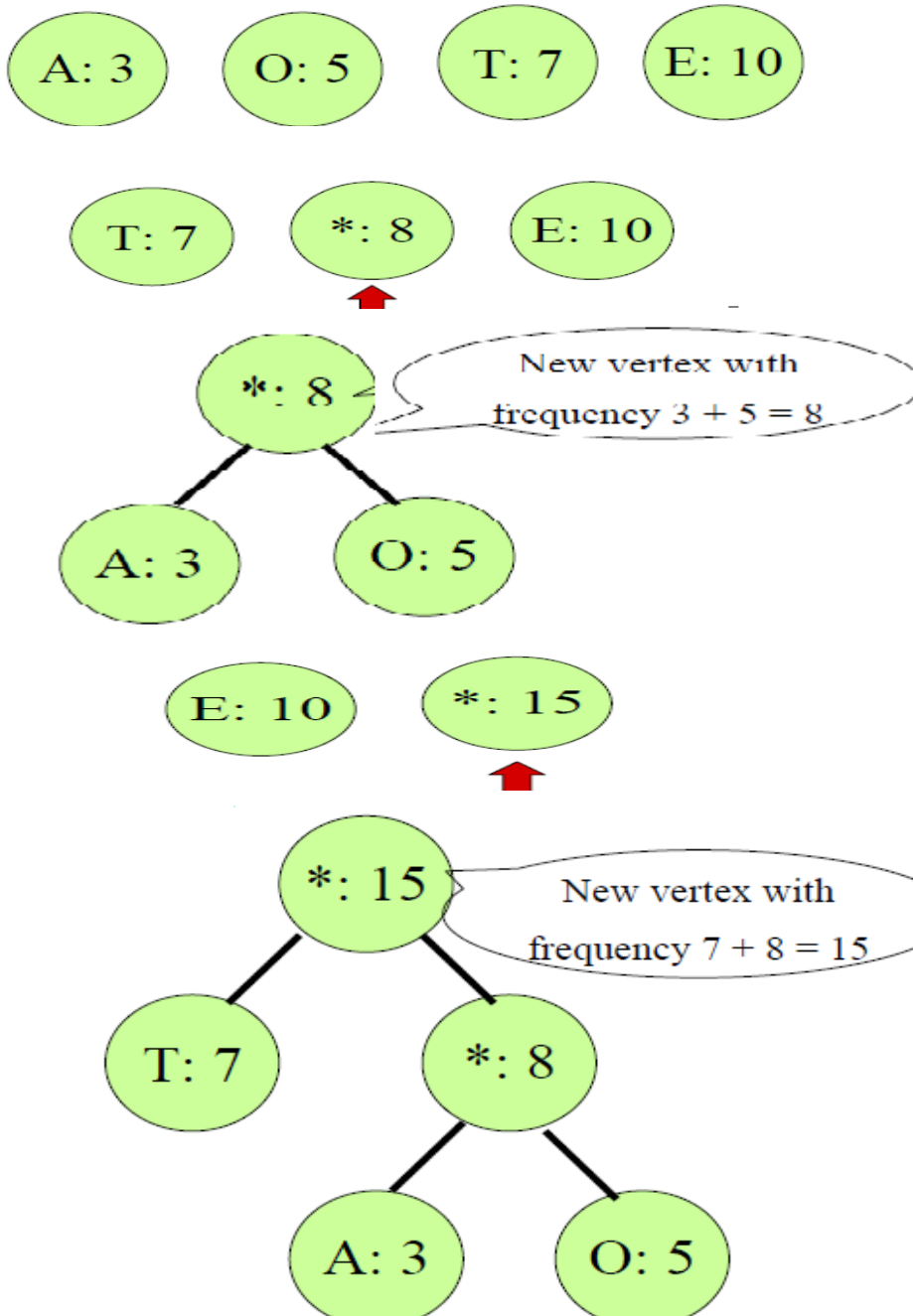
Since to represent 4 characters the 2 bit is sufficient thus take initially two bits for each character this is called fixed length character.

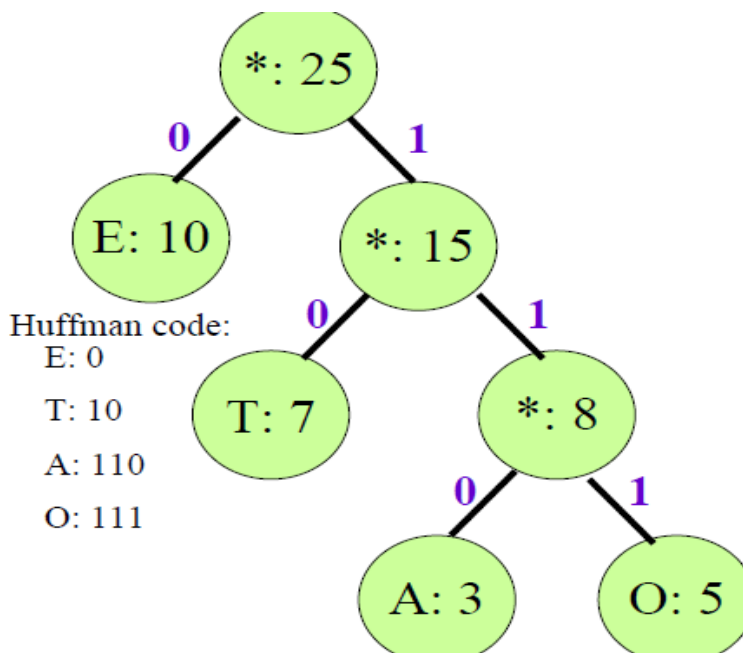
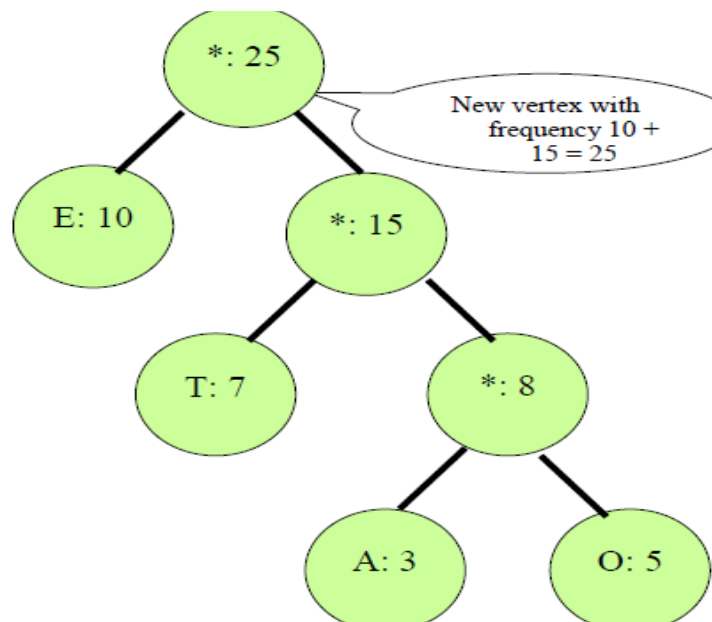
<u>character</u>	<u>frequencies</u>
E:	10
T:	07
O:	05
A:	03

Now sort these characters according to their frequencies in non-decreasing order.

<u>character</u>	<u>frequencies</u>	<u>code</u>
A:	03	00
O:	05	01
T:	07	10
E:	10	11

Here before using Huffman algorithm the total number of bits required is  
 $nb = 3 \times 2 + 5 \times 2 + 7 \times 2 + 10 \times 2 = 06 + 10 + 14 + 20 = 50 \text{ bits}$





Left branch is 0

Right branch is 1

Now from variable length code we get following code sequence.

<u>character</u>	<u>frequencies</u>	<u>code</u>
A:	03	110
O:	05	111
T:	07	10
E:	10	0

Thus after using Huffman algorithm the total number of bits required is

$$nb = 3 \times 3 + 5 \times 3 + 7 \times 2 + 10 \times 1 = 09 + 15 + 14 + 10 = 48 \text{ bits}$$

$$(50 - 48) / 50 \times 100\% = 4\%$$

Since in this small example we save about 4% space by using Huffman algorithm. If we take large example with a lot of characters and their frequencies we can save a lot of space.

## Unit 7

### Sorting:

- a) **Introduction**
- b) **Bubble sort**
- c) **Insertion sort**
- d) **Selection sort**
- e) **Quick sort**
- f) **Merge sort**
- g) **Comparison and efficiency of sorting**

### Introduction:

#### Sorting

Sorting is among the most basic problems in algorithm design. We are given a sequence of items, each associated with a given key value. The problem is to permute the items so that they are in increasing (or decreasing) order by key. Sorting is important because it is often the first step in more complex algorithms. Sorting algorithms are usually divided into two classes, internal sorting algorithms, which assume that data is stored in an array in main memory, and external sorting algorithm, which assume that data is stored on disk or some other device that is best accessed sequentially. We will only consider internal sorting. Sorting algorithms often have additional properties that are of interest, depending on the application. Here are two important properties.

In brief the sorting is a process of arranging the items in a list in some order that is either ascending or descending order.

Let  $a[n]$  be an array of  $n$  elements  $a_0, a_1, a_2, a_3, \dots, a_{n-1}$  in memory. The sorting of the array  $a[n]$  means arranging the content of  $a[n]$  in either increasing or decreasing order.

i.e.  $a_0 \leq a_1 \leq a_2 \leq a_3 \leq \dots \leq a_{n-1}$

**consider a list of values:** 2 ,4 ,6 ,8 ,9 ,1 ,22 ,4 ,77 ,8 ,9

**After sorting the values:** 1, 2, 4, 4, 6, 8, 8,9 , 9 , 22, 77

**In-place:** The algorithm uses no additional array storage, and hence (other than perhaps the system's recursion stack) it is possible to sort very large lists without the need to allocate additional working storage.

**Stable:** A sorting algorithm is stable if two elements that are equal remain in the same relative position after sorting is completed. This is of interest, since in some sorting applications you sort first on one key and then on another. It is nice to know that two items that are equal on the second key, remain sorted on the first key.

## **Bubble Sort:**

The basic idea of this sort is to pass through the array sequentially several times. Each pass consists of comparing each element in the array with its successor (for example  $a[i]$  with  $a[i + 1]$ ) and interchanging the two elements if they are not in the proper order. For example, consider the following array:

**Initially:**

25	57	48	37	12	92	86	33
0	1	2	3	4	5	6	7

**After Pass 1:**

25	48	37	12	57	86	33	92
0	1	2	3	4	5	6	7

**After Pass 2:**

25	37	12	48	57	33	86	92
0	1	2	3	4	5	6	7

**After Pass 3:**

25	12	37	48	33	57	86	92
0	1	2	3	4	5	6	7

**After Pass 4:**

12	25	37	33	48	57	86	92
0	1	2	3	4	5	6	7

**After Pass 5:**

12	25	33	37	48	57	86	92
0	1	2	3	4	5	6	7

**After Pass 6:**

12	25	33	37	48	57	86	92
0	1	2	3	4	5	6	7

**After Pass 7:**

12	25	33	37	48	57	86	92
0	1	2	3	4	5	6	7

Here, we notice that after each pass, an element is placed in its proper order and is not considered in succeeding passes. Furthermore, we need  $n - 1$  passes to sort  $n$  elements.

### **Algorithm**

BubbleSort(A, n)

```
{
    for(i = 0; i < n-1; i++)
    {
        for(j = 0; j < n-i-1; j++)
        {
            if(A[j] > A[j+1])
            {
                temp = A[j];
                A[j] = A[j+1];
                A[j+1] = temp;
            }
        }
    }
}
```

```

    }
}

```

**Time Complexity:**

Inner loop executes for (n-1) times when i=0, (n-2) times when i=1 and so on:

$$\text{Time complexity} = (n-1) + (n-2) + (n-3) + \dots + 2 + 1 \\ = O(n^2)$$

There is no best-case linear time complexity for this algorithm.

**Space Complexity:**

Since no extra space besides 3 variables is needed for sorting

Space complexity =  $O(n)$

**Selection Sort:**

**Idea:** Find the least (or greatest) value in the array, swap it into the leftmost(or rightmost) component (where it belongs), and then forget the leftmost component. Do this repeatedly.

Let  $a[n]$  be a linear array of  $n$  elements. The selection sort works as follows:

**pass 1:** Find the location  $loc$  of the smallest element in the list of  $n$  elements  $a[0]$ ,  $a[1]$ ,  $a[2]$ ,  $a[3]$ , ...,  $a[n-1]$  and then interchange  $a[loc]$  and  $a[0]$ .

**Pass 2:** Find the location  $loc$  of the smallest element in the sub-list of  $n-1$  elements  $a[1]$ ,  $a[2]$ ,  $a[3]$ , ...,  $a[n-1]$  and then interchange  $a[loc]$  and  $a[1]$  such that  $a[0]$ ,  $a[1]$  are sorted.

..... and so on.

Then we will get the sorted list  $a[0] \leq a[1] \leq a[2] \leq a[3] \leq \dots \leq a[n-1]$ .

**Algorithm:**

SelectionSort(A)

```

{
    for( i = 0; i < n ; i++)
    {
        least=A[i];
        p=i;
        for ( j = i + 1; j < n ; j++)
        {
            if (A[j] < A[i])
                least= A[j]; p=j;
        }
        swap(A[i],A[p]);
    }
}

```

**Time Complexity:**

Inner loop executes for (n-1) times when i=0, (n-2) times when i=1 and so on:

$$\text{Time complexity} = (n-1) + (n-2) + (n-3) + \dots + 2 + 1 \\ = O(n^2)$$

There is no best-case linear time complexity for this algorithm, but number of swap operations is reduced greatly.

**Space Complexity:**

Since no extra space besides 5 variables is needed for sorting

Space complexity =  $O(n)$



**Insertion Sort:**

**Idea:** like sorting a hand of playing cards start with an empty left hand and the cards facing down on the table. Remove one card at a time from the table, and insert it into the correct position in the left hand. Compare it with each of the cards already in the hand, from right to left. The cards held in the left hand are sorted

Suppose an array  $a[n]$  with  $n$  elements. The insertion sort works as follows:

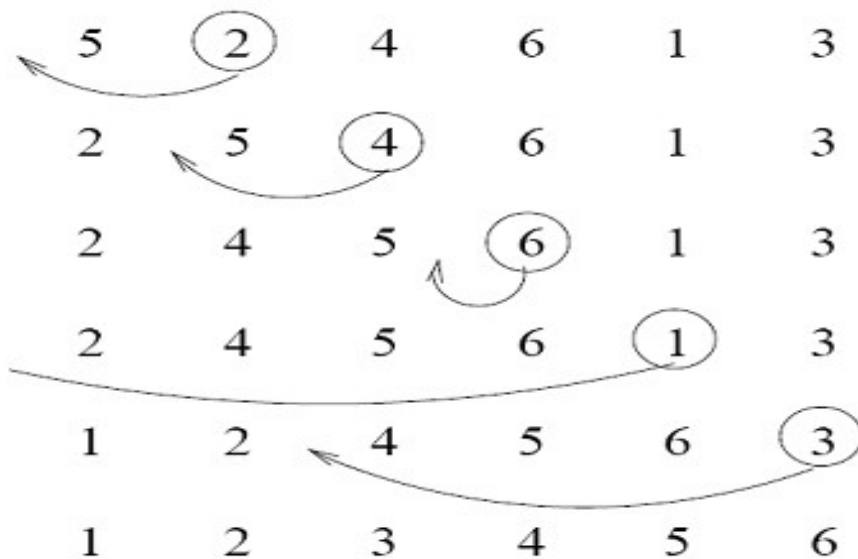
**pass 1:**  $a[0]$  by itself is trivially sorted.

**Pass 2:**  $a[1]$  is inserted either before or after  $a[0]$  so that  $a[0], a[1]$  is sorted.

**Pass 3:**  $a[2]$  is inserted into its proper place in  $a[0], a[1]$  that is before  $a[0]$ , between  $a[0]$  and  $a[1]$ , or after  $a[1]$  so that  $a[0], a[1], a[2]$  is sorted.

.....  
**pass N:**  $a[n-1]$  is inserted into its proper place in  $a[0], a[1], a[2], \dots, a[n-2]$  so that  $a[0], a[1], a[2], \dots, a[n-1]$  is sorted with  $n$  elements.

**Example:**



**Algorithm:**

```

InsertionSort(A)
{
    for (i=1; i<n; i++)
    {
        key = A[i]
        for(j=i; j>0 && A[j] >key; j--)
        {
            A[j+1] = A[j]
        }
        A[j+1] = key
    }
}

```

**Time Complexity:****Worst Case Analysis:**

Array elements are in reverse sorted order

Inner loop executes for 1 times when  $i=1$ , 2 times when  $i=2$ ... and  $n-1$  times when  $i=n-1$ :

$$\text{Time complexity} = 1 + 2 + 3 + \dots + (n-2) + (n-1) \\ = O(n^2)$$

**Best case Analysis:**

Array elements are already sorted

Inner loop executes for 1 times when  $i=1$ , 1 times when  $i=2$ ... and 1 times when  $i=n-1$ :

$$\text{Time complexity} = 1 + 2 + 3 + \dots + 1 + 1 \\ = O(n)$$

**Space Complexity:**

Since no extra space besides 5 variables is needed for sorting

$$\text{Space complexity} = O(n)$$

**Quick Sort:**

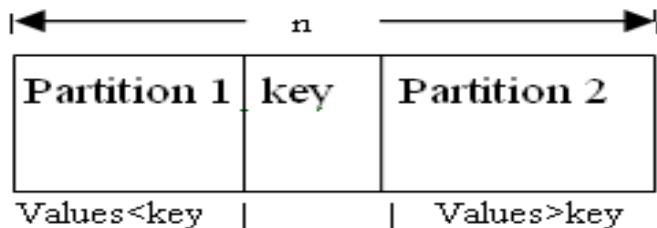
Quick sort developed by C.A.R Hoare is an unstable sorting. In practice this is the fastest sorting method. It possesses very good average case complexity among all the sorting algorithms. This algorithm is based on the divide and conquer paradigm. The main idea behind this sorting is partitioning of the elements.

***Steps for Quick Sort:***

**Divide:** partition the array into two nonempty sub arrays.

**Conquer:** two sub arrays are sorted recursively.

**Combine:** two sub arrays are already sorted in place so no need to combine.



**Example:**  $a[] = \{5, 3, 2, 6, 4, 1, 3, 7\}$

5	3	2	6	4	1	3	7
$x$							$y$

5	3	2	6	4	1	3	7
			$x$			$y$	{swap $x$ & $y$ }

5	3	2	3	4	1	6	7
					$y$	$x$	{swap $y$ and pivot}

1	3	2	3	4	5	6	7
					$p$		

(1 3 2 3 4) 5 (5 7)

and continue this process for each sub-arrays and finally we get a sorted array.

### Algorithm:

QuickSort( $A, l, r$ )

```
{
    f( $l < r$ )
    {
         $p = \text{Partition}(A, l, r);$ 
        QuickSort( $A, l, p-1$ );
        QuickSort( $A, p+1, r$ );
    }
}
```

Partition( $A, l, r$ )

```
{
     $x = l;$ 
     $y = r;$ 
     $p = A[l];$ 
    while( $x < y$ )
    {
        while( $A[x] \leq p$ )
             $x++;$ 
        while( $A[y] \geq p$ )
             $y--;$ 
        if( $x < y$ )
            swap( $A[x], A[y]$ );
    }
}
```

```

    }
    A[l] = A[y];
    A[y] = p;
    return y;      //return position of pivot
}

```

**Time Complexity:****Best Case:**

Divides the array into two partitions of equal size, therefore

$T(n) = 2T(n/2) + O(n)$ , Solving this recurrence we get,

$T(n) = O(n \log n)$

**Worst case:**

when one partition contains the  $n-1$  elements and another partition contains only one element.

Therefore its recurrence relation is:

$T(n) = T(n-1) + O(n)$ , Solving this recurrence we get

$T(n) = O(n^2)$

**Average case:**

Good and bad splits are randomly distributed across throughout the tree

$T_1(n) = 2T'(n/2) + O(n)$  Balanced

$T'(n) = T(n-1) + O(n)$  Unbalanced

Solving:

$B(n) = 2(B(n/2 - 1) + \Theta(n/2)) + \Theta(n)$

$= 2B(n/2 - 1) + \Theta(n)$

$= O(n \log n)$

$\Rightarrow T(n) = O(n \log n)$

**Merge Sort**

To sort an array  $A[l \dots r]$ :

**• Divide**

– Divide the  $n$ -element sequence to be sorted into two sub-sequences of  $n/2$  elements

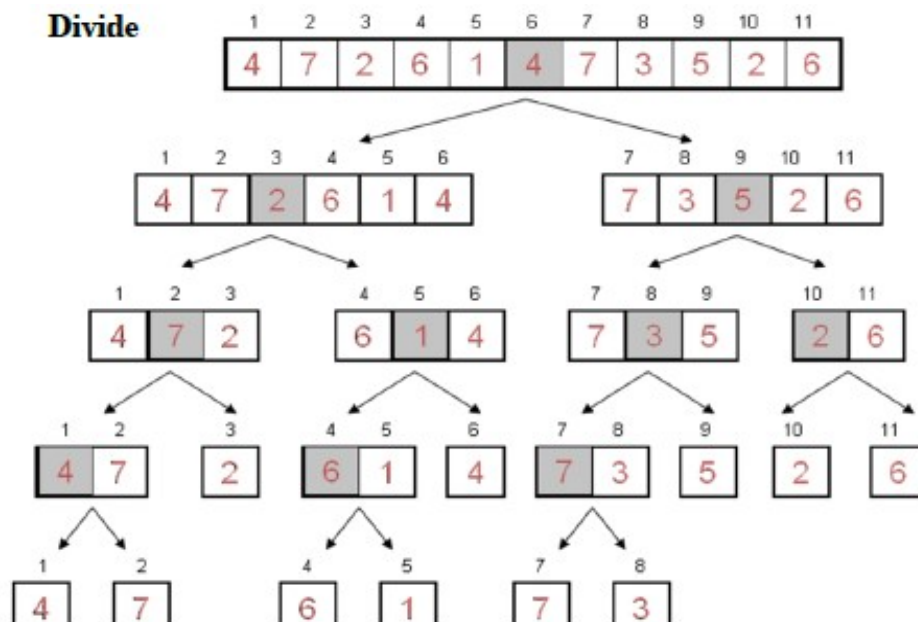
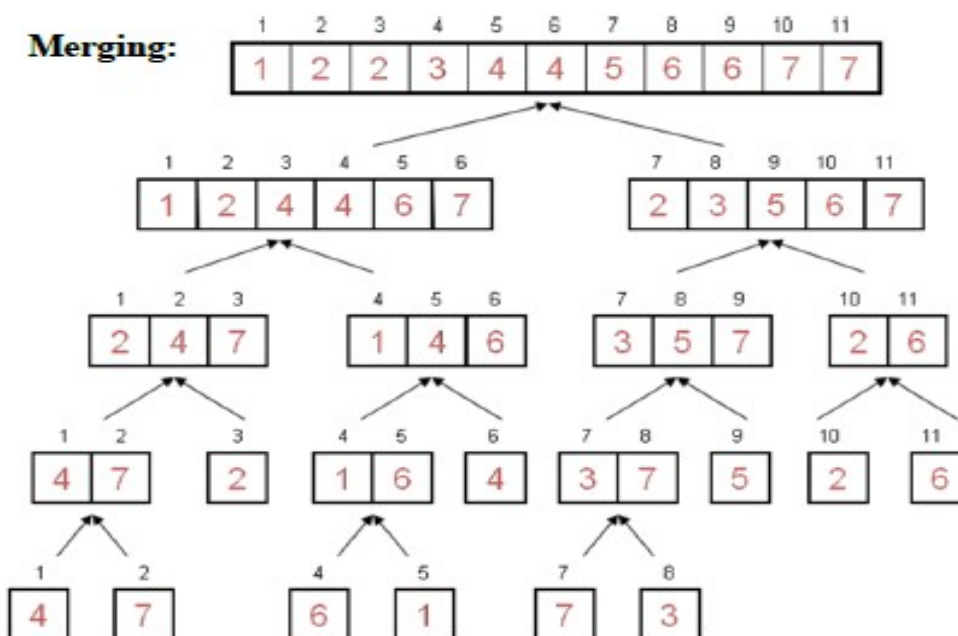
**• Conquer**

– Sort the sub-sequences recursively using merge sort. When the size of the sequences is 1 there is nothing more to do

**• Combine**

Merge the two sorted sub-sequences

Example:  $a[] = \{4, 7, 2, 6, 1, 4, 7, 3, 5, 2, 6\}$

**Divide****Merging:****Algorithm:**

MergeSort(A, 1, r)

{

If (1 &lt; r)

{

    m =  $\lfloor (1 + r) / 2 \rfloor$ 

MergeSort(A, 1, m)

MergeSort(A, m + 1, r)

Merge(A, 1, m + 1, r)

}

}

//Check for base case

//Divide

//Conquer

//Conquer

//Combine

```

Merge(A,B,l,m,r)
{
    x=l, y=m;
    k=l;
    while(x<m && y<r)
    {
        if(A[x] < A[y])
        {
            B[k]= A[x];
            k++; x++;
        }
        else
        {
            B[k] = A[y];
            k++; y++;
        }
    }
    while(x<m)
    {
        A[k] = A[x];
        k++; x++;
    }
    while(y<r)
    {
        A[k] = A[y];
        k++; y++;
    }
    for(i=l; i<= r; i++)
    {
        A[i] = B[i]
    }
}

```

**Time Complexity:**

Recurrence Relation for Merge sort:

$$T(n) = 1 \text{ if } n=1$$

$$T(n) = 2 T(n/2) + O(n) \text{ if } n>1$$

Solving this recurrence we get

$$T(n) = O(n \log n)$$

**Space Complexity:**

It uses one extra array and some extra variables during sorting, therefore

$$\text{Space Complexity} = 2n + c = O(n)$$

## Sorting Comparison:

Sort	Worst Case	Average Case	Best Case	Comments
Insertion Sort	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n)$	
Selection Sort	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	(*Unstable)
Bubble Sort	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	
Merge Sort	$\Theta(n \log n)$	$\Theta(n \log n)$	$\Theta(n \log n)$	Requires Memory
Heap Sort	$\Theta(n \log n)$	$\Theta(n \log n)$	$\Theta(n \log n)$	*Large constants
Quick Sort	$\Theta(n^2)$	$\Theta(n \log n)$	$\Theta(n \log n)$	*Small constants

### Unit 8: Searching:

- Introduction
- Sequential search
- Binary search
- Comparison and efficiency of searching
- Hashing
  - ◆ probing (Linear and Quadratic)

### Introduction:

Searching is a process of finding an element within the list of elements stored in any order or randomly. Searching is divided into two categories Linear and Binary search.

### Sequential Search:

In linear search, access each element of an array one by one sequentially and see whether it is desired element or not. A search will be unsuccessful if all the elements are accessed and the desired element is not found.

In brief, Simply search for the given element left to right and return the index of the element, if found. Otherwise return “Not Found”.

#### **Algorithm:**

```

LinearSearch(A, n, key)
{
    for(i=0; i<n; i++)
    {
        if(A[i] == key)
            return i;
    }
    return -1; // -1 indicates unsuccessful search
}

```

#### **Analysis:**

**Time complexity =  $O(n)$**

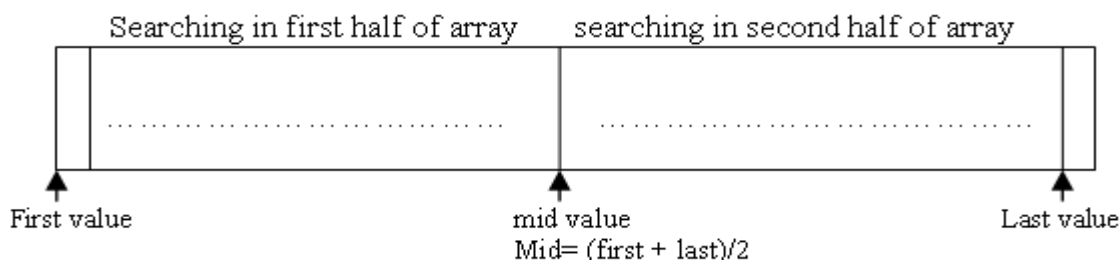
**Binary Search:**

Binary search is an extremely efficient algorithm. This search technique searches the given item in minimum possible comparisons. To do this binary search, first we need to sort the array elements. The logic behind this technique is given below:

- ✓ First find the middle element of the array
- ✓ compare the middle element with an item.
- ✓ There are three cases:
  - ✗ If it is a desired element then search is successful
  - ✗ If it is less than desired item then search only the first half of the array.
  - ✗ If it is greater than the desired element, search in the second half of the array.

Repeat the same process until element is found or exhausts in the search area.

In this algorithm every time we are reducing the search area.

**Running example:**

Take input array  $a[] = \{2, 5, 7, 9, 18, 45, 53, 59, 67, 72, 88, 95, 101, 104\}$

For key = 2

low	high	mid	
0	13	6	key < A[6]
0	5	2	key < A[2]
0	1	0	

Terminating condition, since  $A[mid] = 2$ , return 1(successful).

For key = 103

low	high	mid	
0	13	6	key > A[6]
7	13	10	key > A[10]
11	13	12	key > A[12]
13	13	-	

Terminating condition  $high == low$ , since  $A[0] \neq 103$ , return 0(unsuccessful).

For key = 67

low	high	mid	
0	13	6	key > A[6]
7	13	10	key < A[10]
7	9	8	

Terminating condition, since  $A[mid] = 67$ , return 9(successful).



**Algorithm:**

```

BinarySearch(A,l,r, key)
{
    if(l==r) //only one element
    {
        if(key == A[l])
            return l+1;    //index starts from 0
        else
            return 0;
    }
    else
    {
        m = (l + r) /2 ;    //integer division
        if(key == A[m])
            return m+1;
        else if (key < A[m])
            return BinarySearch(l, m-1, key) ;
        else
            return BinarySearch(m+1, r, key) ;
    }
}

```

**Efficiency:**

From the above algorithm we can say that the running time of the algorithm is

$$T(n) = T(n/2) + O(1)$$

$$= O(\log n) \text{ (verify).}$$

In the best case output is obtained at one run i.e.  $O(1)$  time if the key is at middle.

In the worst case the output is at the end of the array so running time is  $O(\log n)$  time.

In the average case also running time is  $O(\log n)$ .

For unsuccessful search best, worst and average time complexity is  $O(\log n)$ .

**Hashing:**

It is an efficient searching technique in which key is placed in direct accessible address for rapid search.

Hashing provides the direct access of records from the file no matter where the record is in the file. Due to which it reduces the unnecessary comparisons. This technique uses a hashing function say  $h$  which maps the key with the corresponding key address or location.

*A function that transforms a key into a table index is called a hash function.*

A common hash function is

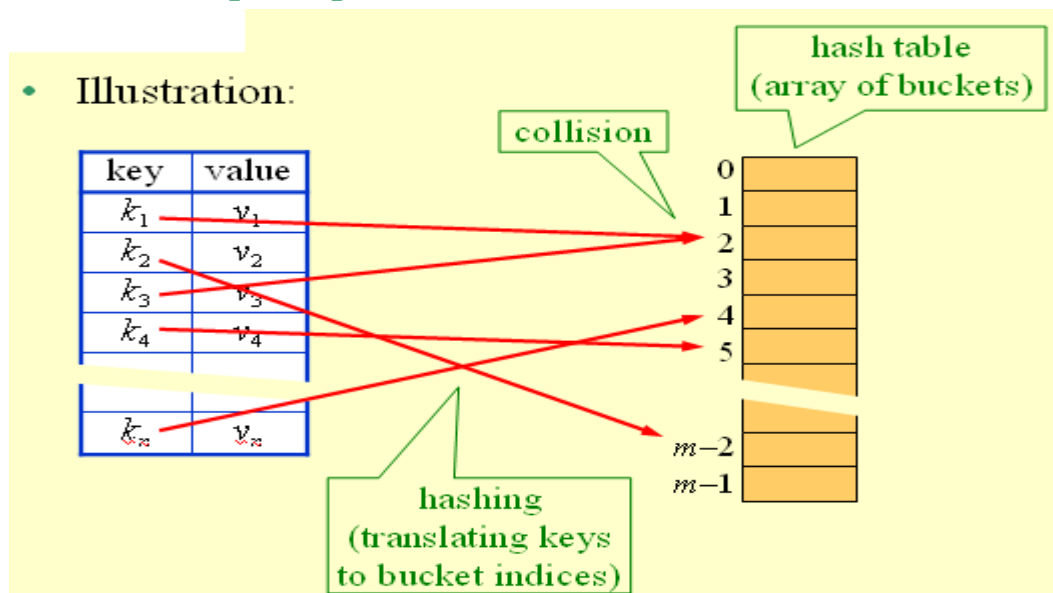
$$h(x) = x \bmod \text{SIZE}$$

if key=27 and SIZE=10 then

$$\text{hash address} = 27 \% 10 = 7$$

## Hash-table principles:

### • Illustration:



### Hash collision:

If two or more than two records trying to insert in a single index of a hash table then such a situation is called hash collision.

Some popular methods for minimizing collision are:

- ✓ Linear probing
- ✓ Quadratic probing
- ✓ Rehashing
- ✓ Chaining
- ✓ Hashing using buckets etc

But here we need only first two methods for minimizing collision

### Linear probing:

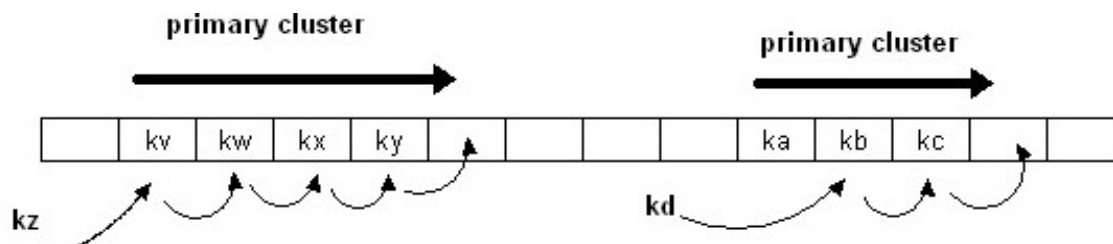
A hash-table in which a collision is resolved by putting the item in the next empty place within the occupied array space.

It starts with a location where the collision occurred and does a sequential search through a hash table for the desired empty location.

Hence this method searches in straight line, and it is therefore called linear probing.

#### Disadvantage:

Clustering problem



#### Example:

Insert keys {89, 18, 49, 58, 69} with the hash function  $h(x) = x \bmod 10$  using linear probing.

solution:

when  $x=89$ :

$$h(89)=89\%10=9$$

insert key 89 in hash-table in location 9

when  $x=18$ :

$$h(18)=18\%10=8$$

insert key 18 in hash-table in location 8

when  $x=49$ :

$$h(49)=49\%10=9 \text{ (Collision occur)}$$

so insert key 49 in hash-table in next possible vacant location of 9 is 0

when  $x=58$ :

$$h(58)=58\%10=8 \text{ (Collision occur)}$$

insert key 58 in hash-table in next possible vacant location of 8 is 1  
(since 9, 0 already contains values).

when  $x=69$ :

$$h(69)=69\%10=9 \text{ (Collision occur)}$$

insert key 69 in hash-table in next possible vacant location of 9 is 2  
(since 0, 1 already contains values).

0	49
1	58
2	69
3	
4	
5	
6	
7	
8	18
9	89

Fig Hash-table for above keys using linear probing

### **Quadratic Probing:**

Quadratic probing is a collision resolution method that eliminates the primary clustering problem take place in a linear probing.

When collision occur then the quadratic probing works as follows:

$$(\text{Hash value} + 1^2)\% \text{ table size}$$

if there is again collision occur then there exist rehashing.

$$(\text{hash value} + 2^2)\% \text{ table size}$$

if there is again collision occur then there exist rehashing.

$$(\text{hash value} + 3^2)\% \text{ table size}$$

in general in  $i$ th collision

$$h_i(x) = (\text{hash value} + i^2)\% \text{ size}$$

**Example:****Insert keys {89, 18, 49, 58, 69} with the hash-table size 10 using quadratic probing.**

solution:

when  $x=89$ :

$$h(89)=89\%10=9$$

insert key 89 in hash-table in location 9

when  $x=18$ :

$$h(18)=18\%10=8$$

insert key 18 in hash-table in location 8

when  $x=49$ :

$$h(49)=49\%10=9 \text{ (Collision occur)}$$

so use following hash function,

$$h_1(49)=(49+1)\%10=0$$

hence insert key 49 in hash-table in location 0

when  $x=58$ :

$$h(58)=58\%10=8 \text{ (Collision occur)}$$

so use following hash function,

$$h_1(58)=(58+1)\%10=9$$

again collision occur use again the following hash function ,

$$h_2(58)=(58+22)\%10=2$$

insert key 58 in hash-table in location 2

when  $x=69$ :

$$h(69)=69\%10=9 \text{ (Collision occur)}$$

so use following hash function,

$$h_1(69)=(69+1)\%10=0$$

again collision occur use again the following hash function ,

$$h_2(69)=(69+22)\%10=3$$

insert key 69 in hash-table in location 3

0	49
1	
2	58
3	69
4	
5	
6	
7	
8	18
9	89

fig:Hash table for above keys using quadratic probing

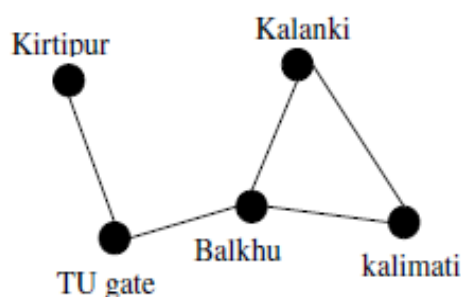
## **Unit:9**

### **Graph:**

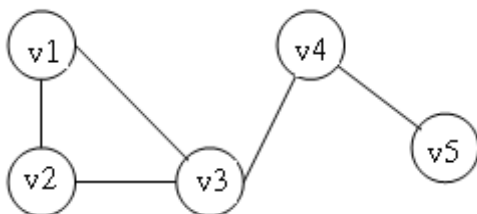
- a) Introduction
- b) Representation of Graph
  - ✓ Array
  - ✓ Linked list
- c) Traversals
  - ✓ Depth first Search
  - ✓ Breadth first search
- d) Minimum spanning tree
  - ✓ Kruskal's algorithm

### **Graph:**

A Graph is a pair  $G = (V, E)$  where  $V$  denotes a set of vertices and  $E$  denotes the set of edges connecting two vertices. Many natural problems can be explained using graph for example modeling road network, electronic circuits, etc. The example below shows the road network.



Let us take a graph:



$$V(G) = \{v1, v2, v3, v4, v5\}$$

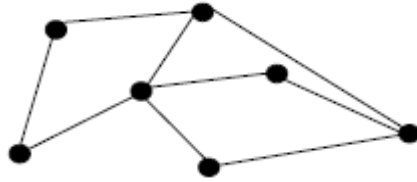
$$E(G) = \{(v1, v2), (v2, v3), (v1, v3), (v3, v4), (v4, v5)\}$$

### **Types of Graph:**

#### **Simple Graph:**

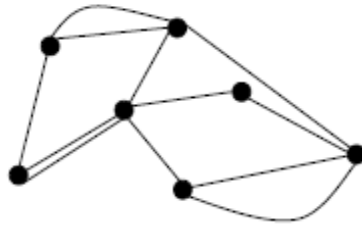
We define a simple graph as 2 – tuple consists of a non empty set of vertices  $V$  and a set of unordered pairs of distinct elements of vertices called edges. We can represent graph as  $G = (V, E)$ . This kind of graph has no loops and can be used for modeling networks that do not have connection to themselves but have both ways connection when two vertices are connected

but no two vertices have more than one connection. The figure below is an example of simple graph.



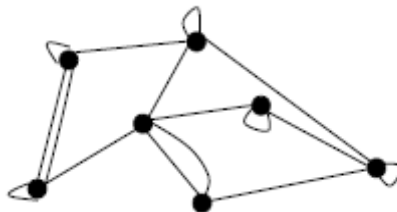
### **Multigraph:**

A multigraph  $G=(V, E)$  consists of a set of vertices  $V$ , a set of edges  $E$ , and a function  $f$  from  $E$  to  $\{\{u, v\} | u, v \in V, u \neq v\}$ . The edges  $e_1$  and  $e_2$  are called multiple or parallel edges if  $f(e_1) = f(e_2)$ . In this representation of graph also loops are not allowed. Since simple graph has single edges every simple graph is a multigraph. The figure below is an example of a multigraph.



### **Pseudograph:**

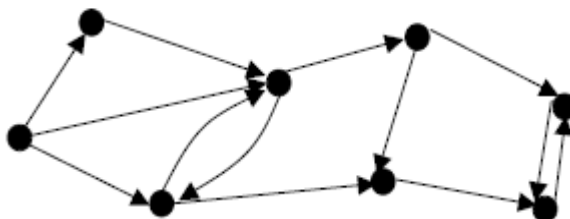
A pseudograph  $G=(V, E)$  consists of a set of vertices  $V$ , a set of edges  $E$ , and a function  $f$  from  $E$  to  $\{\{u, v\} | u, v \in V\}$ . An edge is a loop if  $f(e) = \{u, u\} = \{u\}$  for some  $u \in V$ . The figure below is an example of a multigraph



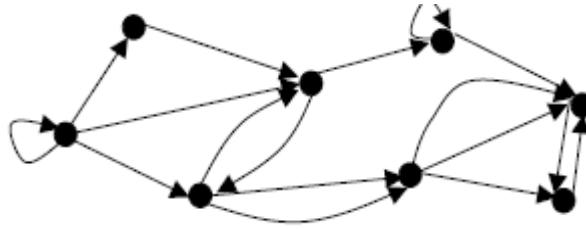
### **Directed Graph:**

A directed graph  $(V, E)$  consists of a set  $V$  of vertices, a set  $E$  of edges that are ordered pairs of elements of  $V$ . The below figure is a directed graph. In this graph loop is allowed but no two vertices can have multiple edges in same direction.

### **Directed Multigraph:**



A directed multigraph  $G = (V, E)$  consists of a set of vertices  $V$ , a set of edges  $E$ , and a function  $f$  from  $E$  to  $\{(u, v) | u, v \in V\}$ . The edges  $e_1$  and  $e_2$  are called multiple edges if  $f(e_1) = f(e_2)$ . The figure below is an example of a directed multigraph.



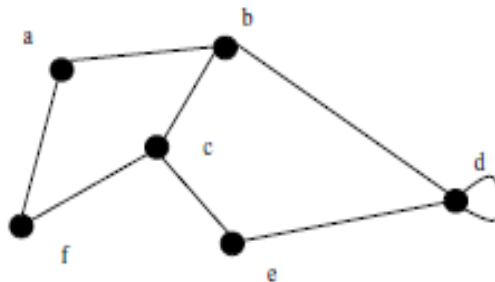
### **Terminologies:**

Two vertices  $u, v$  are adjacent vertices of a graph if  $\{u, v\}$  is an edge.

The edge  $e$  is called incident with the vertices  $u$  and  $v$  if  $e = \{u, v\}$ . This edge is also said to connect  $u$  and  $v$ , where  $u$  and  $v$  are end points of the edge.

**Degree of a vertex** in an undirected graph is the number of edges incident with it, except a loop at a vertex. Loop in a vertex counts twice to the degree. Degree of a vertex  $v$  is denoted by  $\deg(v)$ . A vertex of degree zero is called isolated vertex and a vertex with degree one is called pendant vertex.

**Example:** Find the degrees of the vertices in the following graph.



**Solution:**

$$\deg(a) = \deg(f) = \deg(e) = 2 ; \deg(b) = \deg(c) = 3; \deg(d) = 4$$

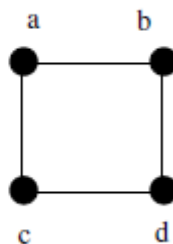
### **Representation of Graph**

Generally graph can be represented in two ways namely adjacency lists (Linked list representation) and adjacency matrix (matrix).

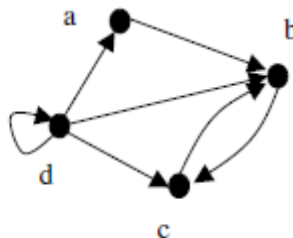
#### **Adjacency List:**

This type of representation is suitable for the undirected graphs without multiple edges, and directed graphs. This representation looks as in the tables below.

Edge List for Simple Graph	
Vertex	Adjacent Vertices
a	b, c
b	a, d
c	a, d
d	b, c



Edge List for Directed Graph	
Initial Vertex	End Vertices
a	b
b	c
c	b
d	a, b, c, d



If we try to apply the algorithms of graph using the representation of graphs by lists of edges, or adjacency lists it can be tedious and time taking if there are high number of edges. For the sake of the computation, the graphs with many edges can be represented in other ways. In this class we discuss two ways of representing graphs in form of matrix.

### **Adjacency Matrix:**

**Given a simple graph  $G=(V, E)$  with  $|V| = n$ . assume that the vertices of the graph are listed in some arbitrary order like  $v_1, v_2, \dots, v_n$ . The adjacency matrix  $A$  of  $G$ , with respect to the order of the vertices is  $n$ -by- $n$  zero-one matrix ( $A = [a_{ij}]$ ) with the condition,**

$$a_{ij} = \begin{cases} 1 & \text{if } \{v_i, v_j\} \text{ is an edge of } G, \\ 0 & \text{otherwise.} \end{cases}$$

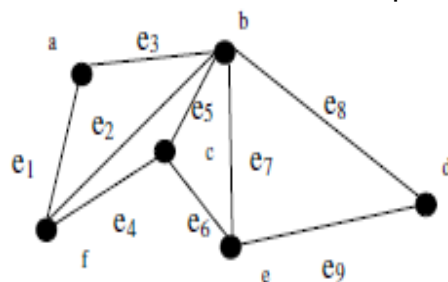
Since there are  $n$  vertices and we may order vertices in any order there are  $n!$  possible order of the vertices. The adjacency matrix depends on the order of the vertices, hence there are  $n!$  possible adjacency matrices for a graph with  $n$  vertices.

In case of the directed graph we can extend the same concept as in undirected graph as dictated by the relation

$$a_{ij} = \begin{cases} 1 & \text{if } (v_i, v_j) \text{ is an edge of } G, \\ 0 & \text{otherwise.} \end{cases}$$

If the number of edges is few then the adjacency matrix becomes sparse. Sometimes it will be beneficial to represented graph with adjacency list in such a condition.



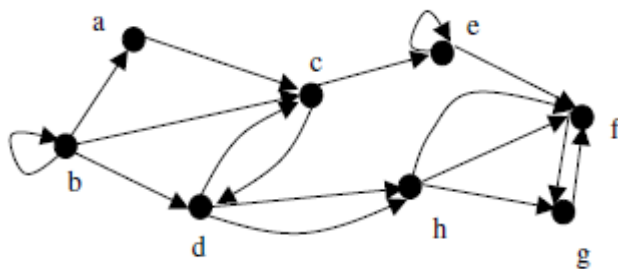


**Solution:** Let the order of the vertices be a, b, c, d, e, f

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Adjacency Matrix

Let us take a directed graph



**Solution:**

Let the order of the vertices be a, b, c, d, e, f, g

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 \end{bmatrix}$$

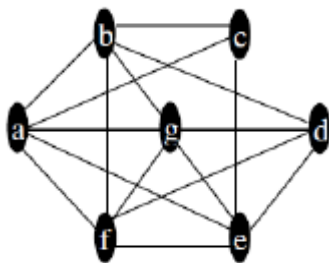
## Graph Traversals

### Breadth-first search:

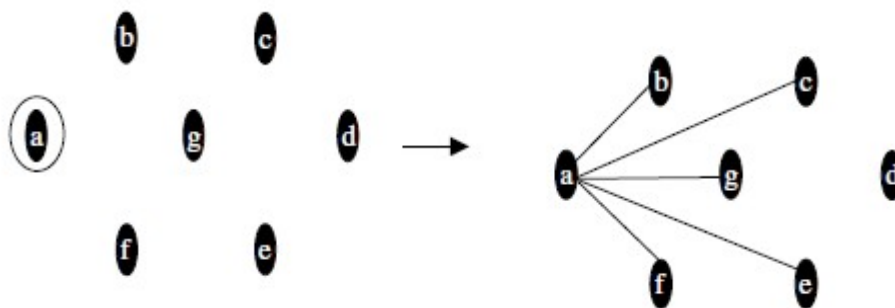
This is one of the simplest methods of graph searching. Choose some vertex arbitrarily as a root. Add all the vertices and edges that are incident in the root. The new vertices added will become the vertices at the level 1 of the BFS tree. Form the set of the added vertices of level 1, find other vertices, such that they are connected by edges at level 1 vertices. Follow the above step until all the vertices are added.

**Example:**

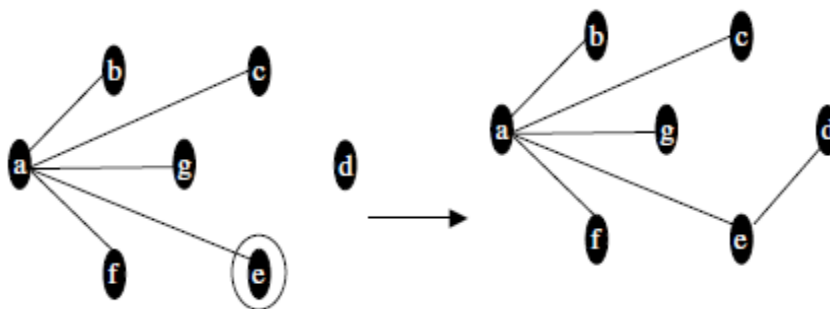
Use breadth first search to find a BFS tree of the following graph

**Solution:**

Choose a as initial vertex then we have



Order the vertices of level 1 i.e. {b, c, g, e, f}. Say order be {e, f, g, b, c}.

**Algorithm:**

BFS(G,s) //s is start vertex

{

$T = \{s\};$

$L = \Phi;$  //an empty queue

  Enqueue(L,s);

  while (  $L \neq \Phi$  )

  {

$v = \text{dequeue}(L);$

    for each neighbor w to v

    if (  $w \notin L$  and  $w \notin T$  )

    {

      enqueue( L,w);

$T = T \cup \{w\};$  //put edge {v,w} also

    }

  }

}

**Analysis**

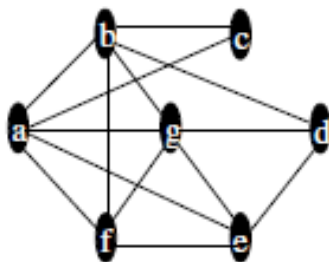
From the algorithm above all the vertices are put once in the queue and they are accessed. For each accessed vertex from the queue their adjacent vertices are looked for and this can be done in  $O(n)$  time (for the worst case the graph is complete). This computation for all the possible vertices that may be in the queue i.e.  $n$ , produce complexity of an algorithm as  $O(n^2)$ .

**Depth First Search:**

This is another technique that can be used to search the graph. Choose a vertex as a root and form a path by starting at a root vertex by successively adding vertices and edges. This process is continued until no possible path can be formed. If the path contains all the vertices then the tree consisting this path is DFS tree. Otherwise, we must add other edges and vertices. For this move back from the last vertex that is met in the previous path and find whether it is possible to find new path starting from the vertex just met. If there is such a path continue the process above. If this cannot be done, move back to another vertex and repeat the process. The whole process is continued until all the vertices are met. This method of search is also called **backtracking**.

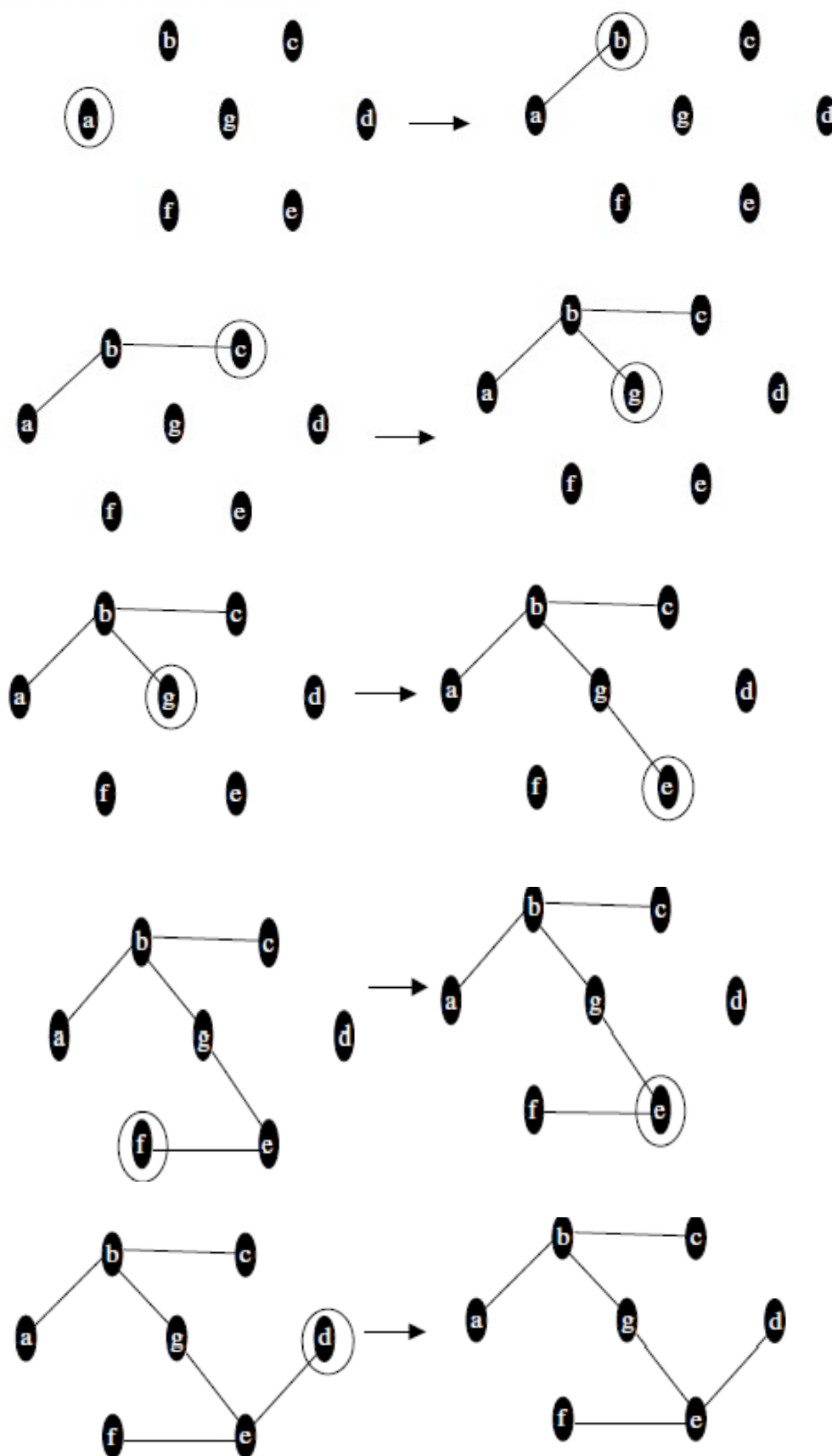
**Example:**

Use depth first search to find a spanning tree of the following graph.



**Solution:**

Choose a as initial vertex then we have



**Algorithm:**

```

DFS(G,s)
{
    T = {s};
    Traverse(s);
}
Traverse(v)
{
    for each w adjacent to v and not yet in T
    {
        T = T U {w}; //put edge {v,w} also
        Traverse (w);
    }
}

```

**Analysis:**

The complexity of the algorithm is greatly affected by **Traverse** function we can write its running time in terms of the relation  $T(n) = T(n-1) + O(n)$ , here  $O(n)$  is for each vertex at most all the vertices are checked (for loop). At each recursive call a vertex is decreased. Solving this we can find that the complexity of an algorithm is  $O(n^2)$ .

**Minimum Spanning Trees**

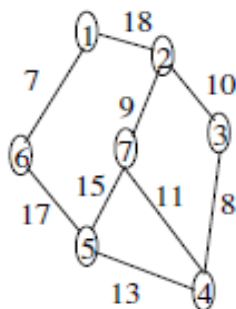
A minimum spanning tree in a connected weighted graph is a spanning tree that has the smallest possible sum of weights of its edges. In this part we study one algorithm that is used to construct the minimum spanning tree from the given connected weighted graph.

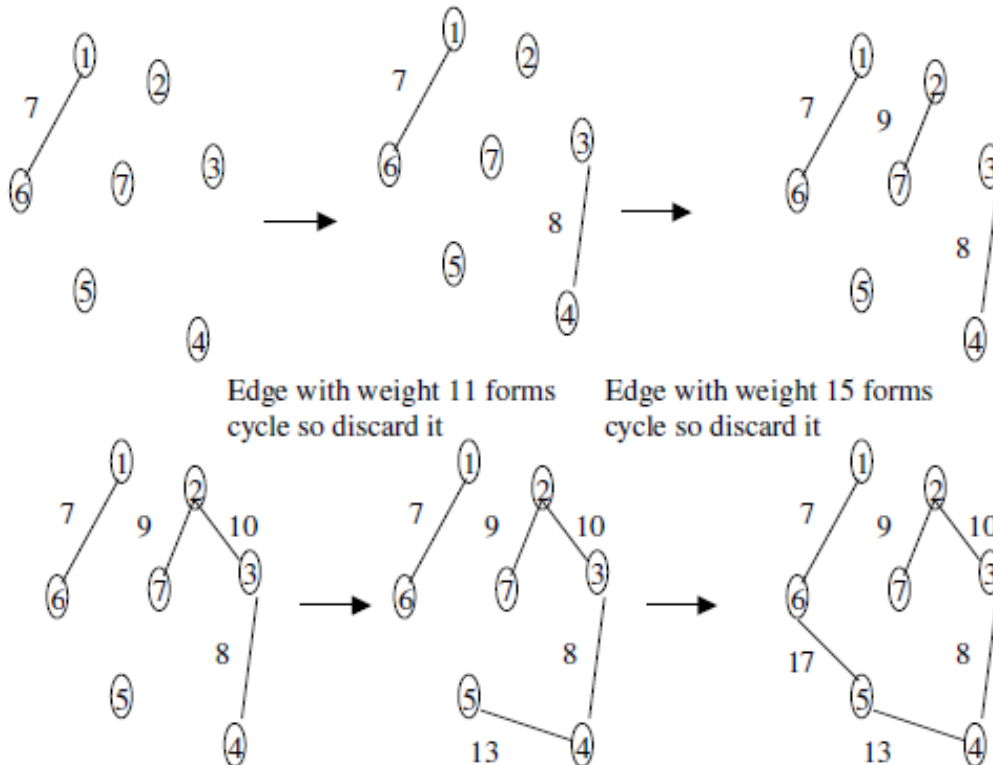
**Kruskal's Algorithm:**

The problem of finding MST can be solved by using Kruskal's algorithm. The idea behind this algorithm is that you put the set of edges from the given graph  $G = (V,E)$  in nondecreasing order of their weights. The selection of each edge in sequence then guarantees that the total cost that would form will be the minimum. Note that we have  $G$  as a graph,  $V$  as a set of  $n$  vertices and  $E$  as set of edges of graph  $G$ .

**Example:**

Find the MST and its weight of the graph.



**Solution:**

The total weight of MST is 64.

**Algorithm:**

*KruskalMST(G)*

```

{
     $T = \{V\}$  // forest of  $n$  nodes
     $S =$  set of edges sorted in nondecreasing order of weight
    while ( $|T| < n-1$  and  $E \neq \emptyset$ )
    {
        Select  $(u,v)$  from  $S$  in order
        Remove  $(u,v)$  from  $E$ 
        if ( $(u,v)$  doesnot create a cycle in  $T$ )
             $T = T \cup \{(u,v)\}$ 
    }
}

```

**Analysis:**

In the above algorithm the  $n$  tree forest at the beginning takes  $(V)$  time, the creation of set  $S$  takes  $O(E \log E)$  time and while loop execute  $O(n)$  times and the steps inside the loop take almost linear time (see disjoint set operations; find and union). So the total time taken is  $O(E \log E)$

# ***A Complete Note in Data Structure and Algorithms***

## ***By Bhupendra Saud***



***Email: [Saud.bhupendra427@gmail.com](mailto:Saud.bhupendra427@gmail.com)***

This Note is prepared for Bsc  
Csit 2<sup>nd</sup> semester students  
according to their course of  
study by **Bhupendra saud**. If  
any defects, errors take place  
please send your views to  
Bhupendra sir or  
[csitnepal.com](http://csitnepal.com)

***coming soon:notes in NM, OS, DAA, C, C++,DBMS.***